

## Recurrence Relations

### Q1.

On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1.5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

- (a) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

Let  $u_n$  and  $u_{n+1}$  represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving  $u_{n+1}$  and  $u_n$ .

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- (b) Find the date and the amount of the final payment.

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### Q2.

A recurrence relation is defined by  $u_{n+1} = pu_n + q$ , where  $-1 < p < 1$  and  $u_0 = 12$ .

- (a) If  $u_1 = 15$  and  $u_2 = 16$ , find the values of  $p$  and  $q$ .

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- (b) Find the limit of this recurrence relation as  $n \rightarrow \infty$ .

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### Q3.

A sequence is defined by the recurrence relation

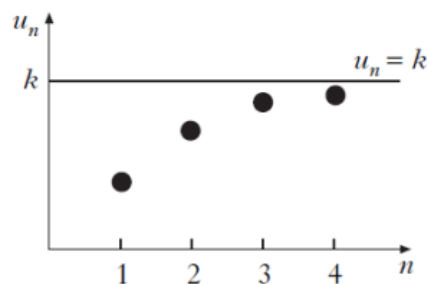
$$u_{n+1} = \frac{1}{4}u_n + 16, \quad u_0 = 0.$$

- (a) Calculate the values of  $u_1$ ,  $u_2$  and  $u_3$ .

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Four terms of this sequence,  $u_1$ ,  $u_2$ ,  $u_3$  and  $u_4$  are plotted as shown in the graph.

As  $n \rightarrow \infty$ , the points on the graph approach the line  $u_n = k$ , where  $k$  is the limit of this sequence.



- (b) (i) Give a reason why this sequence has a limit.

- (ii) Find the exact value of  $k$ .

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**Q4.**

Two sequences are generated by the recurrence relations  $u_{n+1} = au_n + 10$  and  $v_{n+1} = a^2v_n + 16$ .

The two sequences approach the same limit as  $n \rightarrow \infty$ .

Determine the value of  $a$  and evaluate the limit.

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