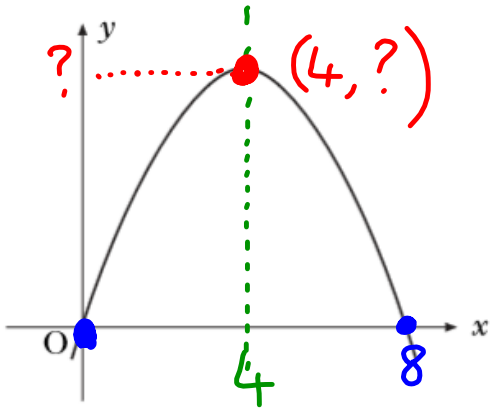
Quadratic Equations & Graphs of Quadratic Functions

1)

The graph shown below is part of the parabola with equation $y = 8x - x^2$.



(*a*) By factorising $8x - x^2$, find the roots of the equation

$$8x - x^2 = 0.$$ **2**

(*b*) State the equation of the axis of symmetry of the parabola. **1**

(*c*) Find the coordinates of the turning point. **2**

a) $8x - x^2 = 0$

$x(8 - x) = 0$ ✓

$x = 0$ or $8 - x = 0$

$x = 8$

$(0, 0)$ & $(8, 0)$ ✓

b) $x = 4$ ✓

c) $x = 4$

$y = 8x - x^2$

$y = 8 \times 4 - 4^2$ ✓

$= 32 - 16$

$= 16$

T.P $(4, 16)$ ✓

2)

Solve the equation

$$2x^2 - 6x - 5 = 0,$$

giving the roots correct to one decimal place. **4**

use the quadratic formula

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 2 \quad b = -6 \quad c = -5$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 2 \times (-5)}}{2 \times 2} \checkmark$$

$$= \frac{6 \pm \sqrt{76}}{4} \checkmark$$

$$x = \frac{6 + \sqrt{76}}{4} \quad \text{or} \quad x = \frac{6 - \sqrt{76}}{4}$$

$= 3 \cdot 679\ldots$ ✓ $= -0 \cdot 679\ldots$

$= 3 \cdot 7$ ✓ $= -0 \cdot 8$

3)

Maria has been asked to find the roots of the equation

$$x^2 + 3x + 5 = 0.$$

She decides to use the quadratic formula

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}.$$

(*a*) Calculate the value of $b^2 - 4ac$. **1**

(*b*) Now explain why Maria cannot find the roots. **1**

a) $a = 1 \quad b = 3 \quad c = 5$

$b^2 - 4ac$

$= 3^2 - 4 \times 1 \times 5$

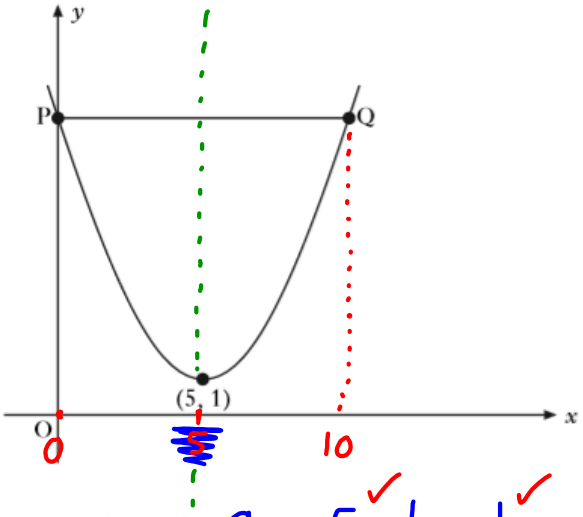
$= -11$ ✓

b) Maria cannot find the roots because there are no real roots since $b^2 - 4ac < 0$ ✓

4)

The graph below shows part of a parabola with equation of the form

$y = (x + a)^2 + b$.



(a) State the values of a and b. $a = -5$ ✓ $b = 1$ ✓ 2

(b) State the equation of the axis of symmetry of the parabola. $x = 5$ ✓ 1

(c) The line PQ is parallel to the x-axis.
Find the coordinates of points P and Q. 3

using symmetry P(0, ?) and Q(10, ?) ✓

$y = (x + a)^2 + b$

$y = (x - 5)^2 + 1$

$x = 0$ $y = (0 - 5)^2 + 1$ ✓

$= (-5)^2 + 1$

$= 25 + 1$

$= 26$

OR $x = 10$

$y = (10 - 5)^2 + 1$

$= 5^2 + 1$

$= 25 + 1$

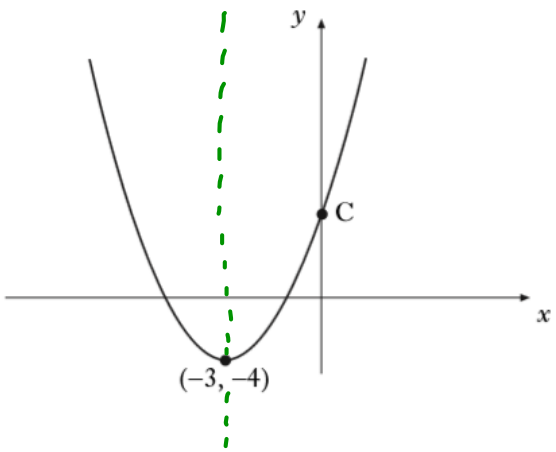
$= 26$

So P is (0, 26) and Q is (10, 26) ✓

5)

The diagram below shows part of a parabola with equation of the form

$$y = (x + a)^2 + b.$$



(*a*) Write down the equation of the axis of symmetry of the graph. **1**

$x = -3$ ✓

(*b*) Write down the equation of the parabola. **2**

(*c*) Find the coordinates of C. **2**

b) Use T.P $(-3, -4)$ $y = (x + a)^2 + b$

$y = (x + 3)^2 - 4$ ✓ ✓

c) C is the y-intercept so $x = 0$ $(0, ?)$

$y = (x + 3)^2 - 4$ (From part b)

$y = (0 + 3)^2 - 4$ ✓

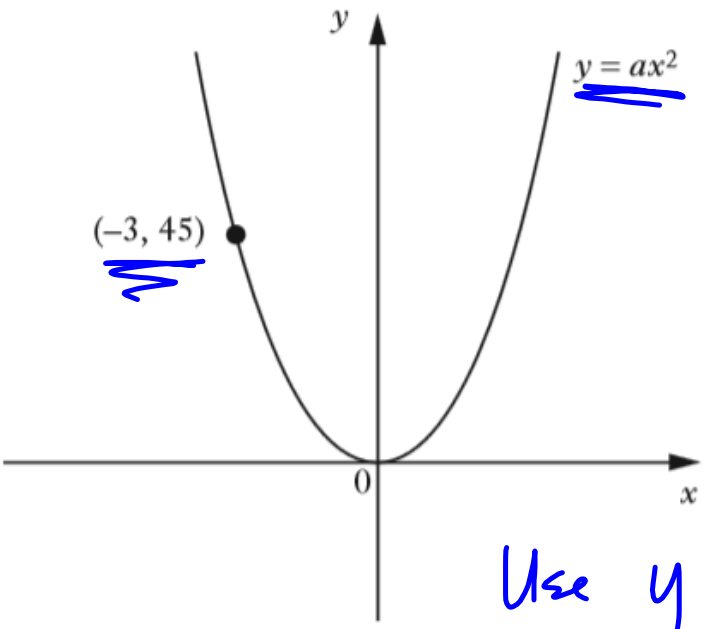
$y = 3^2 - 4$

$y = 9 - 4$

$y = 5$ ✓ So C is $(0, 5)$

6)

The diagram below shows part of the graph of $y = ax^2$.



Find the value of a.

Use $y = ax^2$ and $(-3, 45)$

x y

$45 = a \times (-3)^2$ ✓

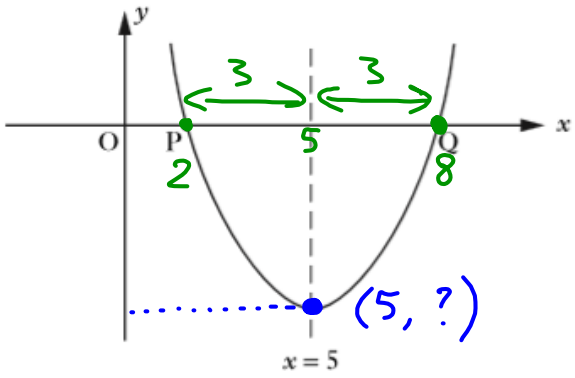
$45 = a \times 9$

$\div 9 \quad \div 9$

$a = 5$ ✓

7)

The graph below shows part of a parabola with equation of the form $y = (x + a)^2 + b$.



The equation of the axis of symmetry of the parabola is $x = 5$.

(a) State the value of a. by using the T.P $a = -5$ ✓ 1

(b) P is the point (2, 0). State the coordinates of Q. 1

(c) Calculate the value of b. 2

b) using symmetry Q is (8, 0) ✓

c) use an equation $y = (x + a)^2 + b$

$y = (x - 5)^2 + b$

and a point (2, 0) or (8, 0)

x y x y

$0 = (2 - 5)^2 + b$ ✓

$0 = (-3)^2 + b$

$0 = 9 + b$

$b = -9$ ✓

(ie, T.P is (5, -9) which looks correct!)

8)

A parabola has equation $y = (x-2)^2 - 5$.

(*a*) Write down the coordinates of the turning point of the parabola. **2**

(*b*) Does this parabola have a maximum or a minimum turning point? **1**

a) Use equation $y = (x-2)^2 - 5$

So T.P is $(2, -5)$ ✓ ✓

b) Minimum T.P ✓ (because 'x^2' is positive so ∪)

9)

Given that

$$x^2 - 10x + 18 = (x - a)^2 + b,$$

find the values of a and b.

3

2

$x^2 - 10x + 18$

$= (x - 5)^2 - 7$ ✓ ✓

$(-5)^2 = 25$

$25 - 7 = 18$

10)

Given that

$$f(x) = x^2 + 3,$$

(a) evaluate $f(-4)$ means $x = -4$ | 2

(b) find t when $f(t) = 52$. means $x = t$ & $f = 52$ | 2

a) $f(x) = x^2 + 3$

$f(-4) = (-4)^2 + 3$ ✓

$= 16 + 3$

$= 19$ ✓

b)

$f(x) = x^2 + 3$

$f(t) = t^2 + 3$

$52 = t^2 + 3$ ✓

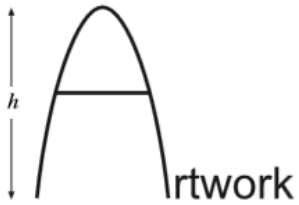
$49 = t^2$

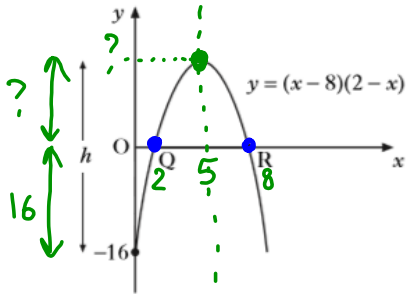
$t = \sqrt{49}$

$= 7$ or -7 ✓

11) The curved part of the letter A in the *Artwork* logo is in the shape of a parabola.

The equation of this parabola is $y = (x - 8)(2 - x)$.





(a) Write down the coordinates of Q and R. (Roots so $y = 0$) 2

(b) Calculate the height, h, of the letter A. (turning point) 3

a) $y = (x - 8)(2 - x)$

$0 = (x - 8)(2 - x)$

$x - 8 = 0$ or $2 - x = 0$

$x = 8$ $x = 2$

Q(2, 0) R(8, 0)

b) Require T.P, using symmetry $x = 5$ ✓

$y = (x - 8)(2 - x)$

$y = (5 - 8)(2 - 5)$

$= -3 \times (-3)$

$= 9$ ✓

So T.P at (5, 9)

Therefore height $= 16 + 9 = 25$ ✓