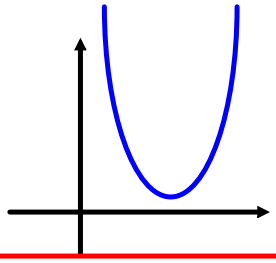
Quadratic Functions & the Discriminant Homework: SOLUTIONS

- if $b^2 - 4ac < 0$



$b^2 - 4ac < 0$

No real roots

1) Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots.

$b^2 - 4ac < 0$ $\quad a = k \quad b = -1 \quad c = -1$

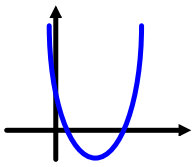
$(-1)^2 - 4 \times k \times (-1) < 0$ ✓

$1 + 4k < 0$

$4k < -1$

$k < -\frac{1}{4}$ ✓

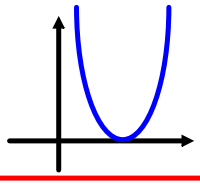
- if $b^2 - 4ac > 0$



$b^2 - 4ac > 0$

2 real & distinct roots

- if $b^2 - 4ac = 0$



$b^2 - 4ac = 0$

2 equal roots

2) Here are two statements about the roots of the equation $x^2 + x + 1 = 0$:

(1) the roots are equal;

(2) the roots are real.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

$a = 1$

$b = 1 \quad c = 1$

$b^2 - 4ac$

$= 1^2 - 4 \times 1 \times 1$

$= 1 - 4$

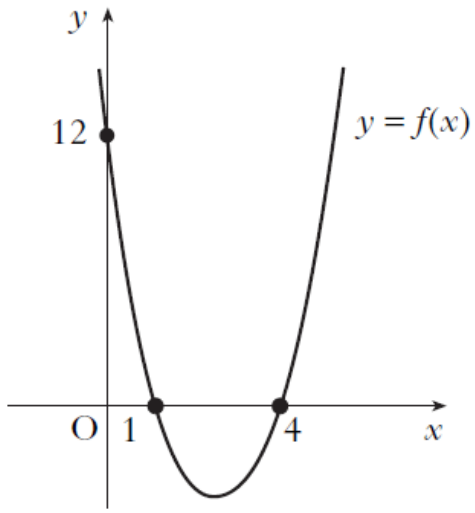
$= -3$ ✓

Roots are not real since $b^2 - 4ac < 0$

So **A** ✓

3) The diagram shows part of the graph of a quadratic function $y = f(x)$.

The graph has an equation of the form $y = k(x - a)(x - b)$.



$y = k(x-a)(x-b)$

Roots $x = 1$ $x = 4$

factors $(x-1)$ $(x-4)$

What is the equation of the graph? So $y = k(x-1)(x-4)$ ✓

find k, use (0, 12) $12 = k(0-1)(0-4)$ ✓

x y $12 = k \times (-1) \times (-4)$

$12 = k \times 4$

$k = 3$

So $y = 3(x-1)(x-4)$ ✓

or $y = 3(x^2 - 4x - x + 4)$

$= 3(x^2 - 5x + 4)$

$= 3x^2 - 15x + 12$

4) $2x^2 + 4x + 7$ is expressed in the form $2(x + p)^2 + q$.

What is the value of q?

$2x^2 + 4x + 7$

$= 2(x^2 + 2x) + 7$ ✓

$= 2(x + 1)^2 + 5$ ✓

So $q = 5$ ✓

5) A function f is given by $f(x) = 2x^2 - x - 9$.

Which of the following describes the nature of the roots of $f(x) = 0$?

A No real roots

B Equal roots

C Real distinct roots

D Rational distinct roots

$2x^2 - x - 9 = 0$

$a = 2 \quad b = -1 \quad c = -9$

$b^2 - 4ac$

$= (-1)^2 - 4 \times 2 \times (-9)$

$= 1 + 72$

$= 73$ ✓

Roots are real + distinct since $b^2 - 4ac > 0$

So (C) ✓

6) For what values of x is $6 + x - x^2 < 0$?

$6 + x - x^2 < 0$

Roots

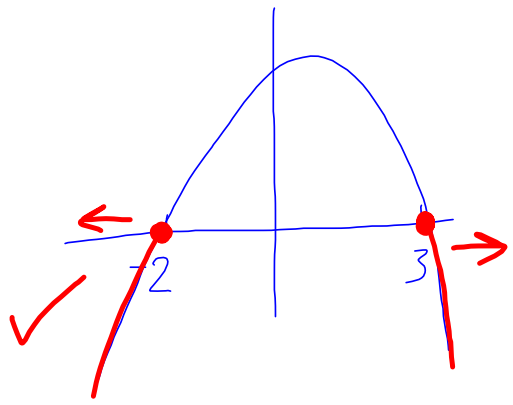
$6 + x - x^2 = 0$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or $x = -2$

Graph



(Solve inequality using graph)

$6 + x - x^2 < 0$ (ie $y < 0$)

$x < -2$ and $x > 3$ ✓

7) The roots of the equation $kx^2 - 3x + 2 = 0$ are equal.
What is the value of k?

$b^2 - 4ac = 0$ $\quad a = k \quad b = -3 \quad c = 2$

$(-3)^2 - 4 \times k \times 2 = 0$ ✓

$9 - 8k = 0$

$8k = 9$

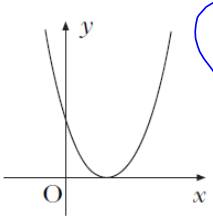
$k = \frac{9}{8}$ ✓

8)

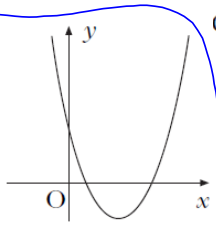
Which of the following diagrams shows a parabola with equation $y = ax^2 + bx + c$, where

- $a > 0$ So happy parabola (A or B) ✓
- $b^2 - 4ac > 0$? So 2 real and distinct roots ✓

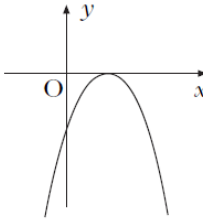
A



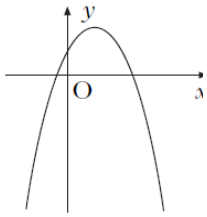
B



C



D



B

TOTAL = 18 marks