Polynomials Homework: SOLUTIONS

1) (*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

(*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

a)

$$\begin{array}{r|rrrr} -2 & 2 & 1 & k & 2 \\ & & -4 & 6 & -2k-12 \\ \hline & 2 & -3 & k+6 & \boxed{-2k-10} \end{array}$$ ✓

So $-2k - 10 = 0$ ✓

$-2k = 10$

$k = -5$ ✓

b) $k = -5$

$2x^3 + x^2 - 5x + 2 = 0$

$(x+2)(2x^2 - 3x + 1) = 0$ ✓

$(x+2)(2x-1)(x-1) = 0$

$x = -2 \quad x = \frac{1}{2} \quad x = 1$ ✓

2) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

Try $x = 2$ ie, Divisor is $x - 2$

$$\begin{array}{c|cccc} 2 & 2 & -7 & 4 & 4 \\ & & 4 & -6 & -4 \\ \hline & 2 & -3 & -2 & \boxed{0} \end{array}$$

So $x - 2$ is a factor since the remainder is 0.

$$2x^3 - 7x^2 + 4x + 4$$

$$= (x-2)(2x^2 - 3x - 2)$$ ✓

$$= (x-2)(2x+1)(x-2)$$ ✓

or $$= (x-2)^2(2x+1)$$

3)

Given that $(x - 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d. **5**

$$\begin{array}{r|rrrr} 2 & 3 & 2 & c & d \\ & & 6 & 16 & 2c+32 \\ \hline & 3 & 8 & c+16 & \boxed{2c+d+32} \end{array} \qquad \begin{array}{r|rrrr} -3 & 3 & 2 & c & d \\ & & -9 & 21 & -3c-63 \\ \hline & 3 & -7 & c+21 & \boxed{-3c+d-63} \end{array}$$

So $2c + d + 32 = 0$ (1) So $-3c + d - 63 = 0$ (2)

$$\begin{array}{rl} & 2c + d + 32 = 0 \\ - & -3c + d - 63 = 0 \\ \hline - & 5c \quad + 95 = 0 \end{array}$$

$5c = -95$

$c = -19$

<u>find d</u>

$2c + d + 32 = 0$

$-38 + d + 32 = 0$

$d - 6 = 0$

$\underline{\underline{d = 6}}$

4)

$f(x) = x^3 - x^2 - 5x - 3.$

(*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$.

(ii) Hence or otherwise factorise $f(x)$ fully. **5**

(*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis.

Write down the coordinates of this turning point. **1**

a) i)

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -5 & -3 \\ & & -1 & 2 & 3 \\ \hline & 1 & -2 & -3 & \boxed{0} \end{array}$$

So $(x+1)$ is a factor since the remainder is zero.

ii) $f(x) = x^3 - x^2 - 5x - 3$

$= (x+1)(x^2 - 2x - 3)$

$= (x+1)(x-3)(x+1)$

or $= (x+1)^2(x-3)$

b) $(-1, 0)$

(because of the repeated factor $(x+1)$ which leads to a repeated root, $x = -1$)

5) A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

(i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

i)

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \end{array}$$

$(x-1)$ is a factor since the remainder is 0.

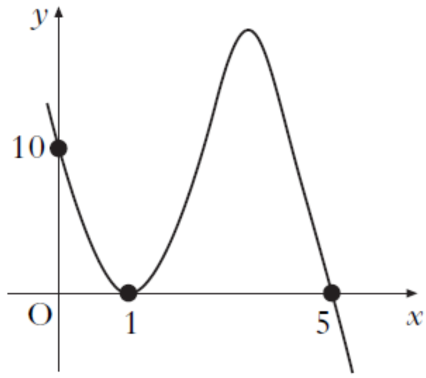
ii) $x^3 - 3x + 2$

$= (x-1)(x^2 + x - 2)$

$= (x-1)(x+2)(x-1)$

or $= (x-1)^2(x+2)$

6) The diagram shows the graph with equation $y = k(x-1)^2(x+t)$.



What are the values of k and t? **2**

$y = k(x-1)^2(x+t)$

Roots $x=1$ $x=1$ $x=5$

Factors $(x-1)$ $(x-1)$ $(x-5)$

T.P = Root so repeated root & repeated factor So $t=-5$ ✓

find k use $(0, 10)$, $x = 0$, $y = 10$

$$y = k(x-1)^2(x+t)$$

$$y = k(x-1)^2(x-5)$$

$$10 = k(0-1)^2(0-5)$$

$$10 = k \times 1 \times (-5)$$

$$10 = -5k$$

$$k = -2$$ ✓

7) A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

What is the remainder when $f(x)$ is divided by $(x - 1)$? **2**

$$
\begin{array}{r|rrrr}
1 & 1 & -1 & 1 & 3 \\
  &   & 1 & 0 & 1 \\
\hline
  & 1 & 0 & 1 & \boxed{4}
\end{array}
$$

So the remainder is 4.

8) On dividing $f(x)$ by $(x - 1)$, the remainder is zero and the quotient is $x^2 - 4x - 5$.

Find $f(x)$ in its fully factorised form. **2**

$$f(x) = (x-1)(x^2-4x-5) \checkmark$$

$$= (x-1)(x-5)(x+1) \checkmark$$