Part 1: Non-Calculator

1)

Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

$\log_a x + \log_a y = \log_a(xy)$	1
$\log_a x - \log_a y = \log_a(\frac{x}{y})$	2
$\log_a x^n = n\log_a x$	3
$\log_a a = 1$	4
$\log_a 1 = 0$	5

$\log_5 2 + \log_5 50 - \log_5 4$

$= \log_5 100 - \log_5 4$ ✓

$= \log_5 25$ ✓

$= 2$ ✓

$\log_5 25 = x$

So $5^x = 25$

$x = 2$

2)

Find x if $4\log_x 6 - 2\log_x 4 = 1$. **3**

$\log_a x + \log_a y = \log_a(xy)$	1
$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$	2
$\log_a x^n = n\log_a x$	3
$\log_a a = 1$	4
$\log_a 1 = 0$	5

$4\log_x 6 - 2\log_x 4 = 1$

$\log_x 6^4 - \log_x 4^2 = 1$

$\log_x\left(\frac{6^4}{4^2}\right) = 1$

$\log_x\left(\frac{1296}{16}\right) = 1$

$\log_x 81 = 1$

$x^1 = 81$

So $\underline{\underline{x = 81}}$

$6^4 = 6 \times 6 \times 6 \times 6$

$= 36 \times 36$

$= 1296$

$36 \times 30 = 108 \times 10 = 1080$

$36 \times 6 = 216$

$$\begin{array}{r} 36 \\ \times\ 6 \\ \hline 216 \\ \hline {}_3 \end{array} \qquad \begin{array}{r} 1080 \\ +\ 216 \\ \hline 1296 \\ \hline \end{array}$$

$\frac{1296}{16} = \frac{648}{8} = \frac{81}{1} = 81$

2)

Find x if $4\log_x 6 - 2\log_x 4 = 1$. **3**

$\log_a x + \log_a y = \log_a(xy)$	1
$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$	2 ✓
$\log_a x^n = n\log_a x$	3 ✓
$\log_a a = 1$	4
$\log_a 1 = 0$	5

$$4\log_x 6 - 2\log_x 4 = 1$$

$$2\log_x 6 - \log_x 4 = \frac{1}{2}$$

$$\log_x 6^2 - \log_x 4 = \frac{1}{2} \checkmark$$

$$\log_x \frac{36}{4} = \frac{1}{2} \checkmark$$

$$\log_x 9 = \frac{1}{2}$$

$$x^{\frac{1}{2}} = 9$$

$$\sqrt{x} = 9$$

sq sq

$$\underline{\underline{x = 81}} \checkmark$$

3)

Solve the equation $\log_2(x+1) - 2\log_2(3) = 3$.

$\log_a x + \log_a y = \log_a(xy)$	1
$\log_a x - \log_a y = \log_a(\frac{x}{y})$	2 ✓
$\log_a x^n = n\log_a x$	3 ✓
$\log_a a = 1$	4
$\log_a 1 = 0$	5

4

$$\log_2(x+1) - 2\log_2 3 = 3$$

$$\log_2(x+1) - \log_2 3^2 = 3 \quad ✓$$

$$\log_2 \frac{(x+1)}{9} = 3 \quad ✓$$

$$2^3 = \frac{(x+1)}{9} \quad ✓$$

$\times 9 \qquad \times 9$

$$72 = x + 1$$

$-1 \qquad -1$

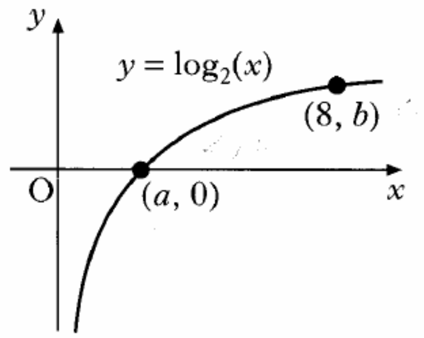
$$\underline{\underline{x = 71}} \quad ✓$$

4)

The diagram shows a sketch of part of the graph of $y = \log_2(x)$.

(*a*) State the values of *a* and *b*. **1**

(*b*) Sketch the graph of $y = \log_2(x + 1) - 3$. **3**



a) $a = 1$

find b use $(8, b)$ — x, y

$y = \log_2 x$

$b = \log_2 8$

$b = 3$

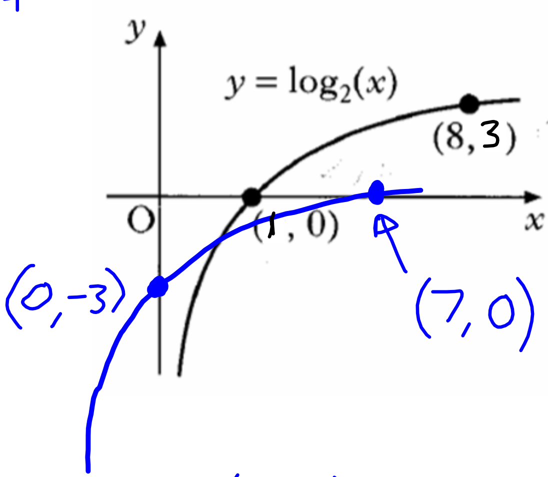
✓

$\left(\log_2 8 = b \quad 2^b = 8 \quad \text{So } 2^3 = 8 \right)$

b) $y = \log_2(x+1) - 3$

Shift 1 to the left

move down by 3



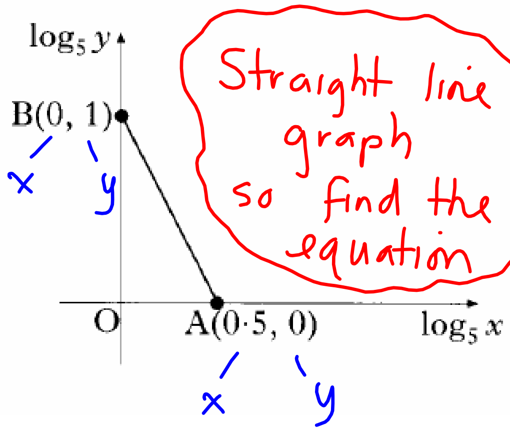
✓ root

✓ y-intercept

✓ correct shape

5)

The graph illustrates the law $y = kx^n$.
If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n. (solve an equation)



Straight line graph so find the equation

4

$y = mx + c$

$m = \dfrac{0-1}{0.5-0} = \dfrac{-1}{0.5} = \dfrac{-2}{1} = -2$

$y = -2x + 1$

So $\log_5 y = -2\log_5 x + 1$ ✓

$\log_5 y + 2\log_5 x = 1$

$\log_5 y + \log_5 x^2 = 1$ ✓

$\log_5 yx^2 = 1$

$5 = yx^2$ ✓

$y = \dfrac{5}{x^2}$

$\log_a x + \log_a y = \log_a(xy)$	1 ✓
$\log_a x - \log_a y = \log_a(\frac{x}{y})$	2
$\log_a x^n = n\log_a x$	3 ✓
$\log_a a = 1$	4 ✓
$\log_a 1 = 0$	5

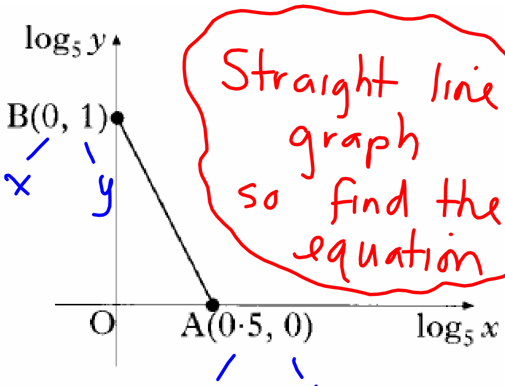
So $y = 5x^{-2}$ ✓

compare with $y = kx^n$, so $k = 5$ & $n = -2$

5)

The graph illustrates the law $y = kx^n$.
If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n. (solve an equation)

4



Straight line graph so find the equation

$$y = mx + c$$

$$y = -2x + 1$$

$$m = \frac{0-1}{0.5-0} = \frac{-1}{0.5} = \frac{-2}{1} = -2$$

So $\log_5 y = -2\log_5 x + 1$

$$\log_5 y = -2\log_5 x + \log_5 5$$

$$\log_5 y = \log_5 x^{-2} + \log_5 5$$

$$\log_5 y = \log_5 5x^{-2}$$

$\log_a x + \log_a y = \log_a(xy)$	1
$\log_a x - \log_a y = \log_a(\frac{x}{y})$	2
$\log_a x^n = n\log_a x$	3
$\log_a a = 1$	4
$\log_a 1 = 0$	5

So $y = 5x^{-2}$

$y = kx^n$ So $k = 5$ and $n = -2$

Part 2: Calculator

6)

Solve an equation

Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3(x-2)+1$ intersects the x-axis. **3**

Root so $y=0$

$$y = \log_3(x-2)+1$$

$$0 = \log_3(x-2)+1$$

-1 -1

$$\log_3(x-2) = -1$$

$$3^{-1} = x-2$$

$$x-2 = \frac{1}{3}$$

$$x = 2\tfrac{1}{3} \text{ or } \frac{7}{3}$$

7)

Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and half hours for the area of the forest fire to double, find the value of the constant k. **3**

$$A = A_0 e^{kt}$$

$t = 1.5$

Let $A_0 = 5$

So $A = 10$ (after 1.5 hours)

$$10 = 5e^{1.5k}$$

$\div 5 \quad \div 5$

$$e^{1.5k} = 2 \quad \checkmark$$

ln ln

$$1.5k = \ln 2 \quad \checkmark$$

$\div 1.5 \quad \div 1.5$

$$k = \ln 2 \div 1.5$$

$$= 0.462 \quad \checkmark$$