Calculating the enclosed area between 2 Graphs

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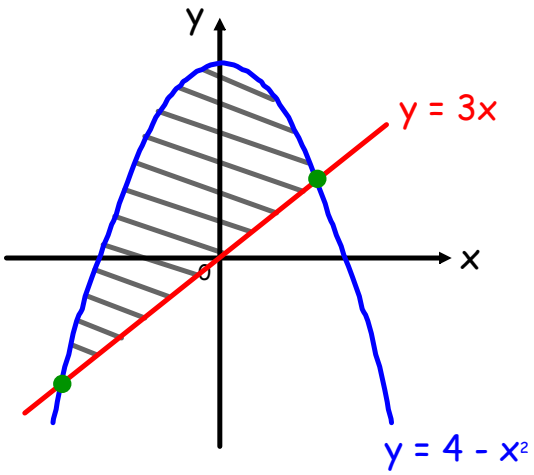
This shaded area is fully enclosed area between 2 Graphs



Calculating the <u>enclosed area</u> between 2 Graphs

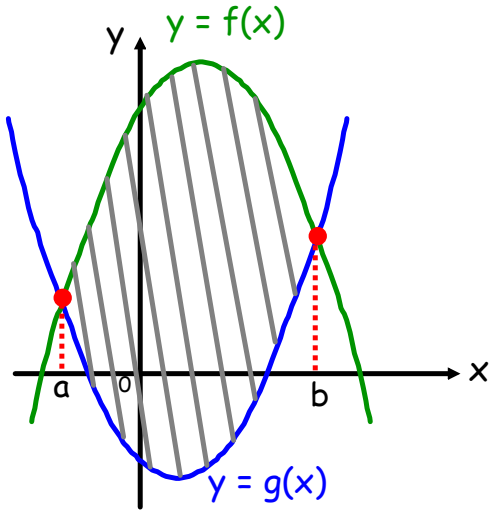
The area enclosed between 2 graphs $y = f(x)$ and $y = g(x)$

from $x = a$ to $x = b$ is given by the Integral:

$$A = \int_a^b (f(x) - g(x)) \, dx$$

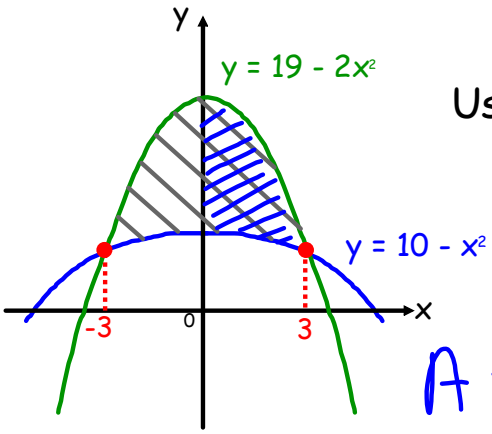
Graph on top - Graph below

(Upper - Lower)



- the integral is the difference of the two functions. You **must** simplify this **before** integrating.
- the limits of integration are the x-cordinates of the points of intersection between the two graphs.
- use symmetry to ease the calculations.

1) Calculate the shaded area.



Use symmetry to ease calculations:

$$A = \int_0^3 19 - 2x^2 - (10 - x^2)\,dx$$

simplify this algebraic expression to ease the integrating

$$= \int_0^3 (9 - x^2)\,dx$$

$$= \left[9x - \frac{x^3}{3}\right]_0^3$$

$$= 9 \times 3 - \frac{3^3}{3} - \left(9 \times 0 - \frac{0^3}{3}\right)$$

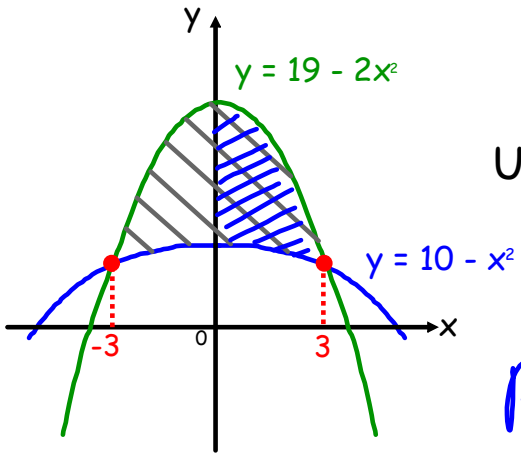
$$= 27 - 9 - 0$$

$$= 18$$

Total Area $= 2 \times 18$

$= 36$ sq units

1) Calculate the shaded area.



OR

Use symmetry to ease calculations:

$$A = 2\int_0^3 19 - 2x^2 - (10 - x^2)\,dx$$

simplify this algebraic expression to ease the integrating

$$= 2\int_0^3 (9 - x^2)\,dx$$

$$= 2\left[9x - \frac{x^3}{3}\right]_0^3$$

$$= 2 \times \left(9 \times 3 - \frac{3^3}{3} - \left(9 \times 0 - \frac{0^3}{3}\right)\right)$$

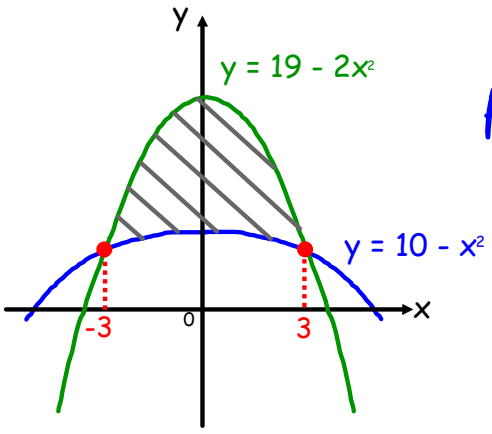
$$= 2 \times (27 - 9 - 0)$$

$$= 2 \times 18$$

$$= 36 \text{ sq units}$$

1) Calculate the shaded area.

Not using symmetry to ease calculations:



$$A = \int_{-3}^{3} 19 - 2x^2 - (10 - x^2)\,dx$$

simplify this algebraic expression to ease the integrating

$$= \int_{-3}^{3} 19 - 2x^2 - 10 + x^2\,dx$$

simplify this algebraic expression to ease the integratin

$$= \int_{-3}^{3} 9 - x^2\,dx$$

$$= \left[9x - \frac{x^3}{3}\right]_{-3}^{3}$$

$$= 9 \times 3 - \frac{3^3}{3} - \left(9 \times -3 - \frac{(-3)^3}{3}\right)$$

$$= 27 - 9 - \left(-27 - \frac{-27}{3}\right)$$

$$= 18 - (-27 + 9)$$

$$= 18 - (-18)$$

$$= 36 \text{ sq units}$$

Calculating the enclosed area between 2 Graphs

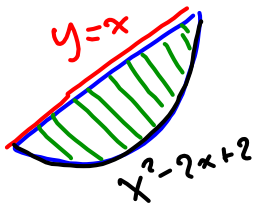
p178 Ex 9O Q1(a & c)

1) a) $A = \int_1^2 x - (x^2 - 2x + 2)\, dx$

simplify this algebraic expression to ease the integrating

$= \int_1^2 x - x^2 + 2x - 2\, dx$

simplify this algebraic expression to ease the integrating



$= \int_1^2 (3x - x^2 - 2)\, dx$

$= \left[\frac{3x^2}{2} - \frac{1}{3}x^3 - 2x\right]_1^2$

$= \left(\frac{3}{2}\times 2^2 - \frac{1}{3}\times 2^3 - 2\times 2\right) - \left(\frac{3}{2}\times 1^2 - \frac{1}{3}\times 1^3 - 2\times 1\right)$ ✓

$= 6 - \frac{8}{3} - 4 - \left(\frac{3}{2} - \frac{1}{3} - 2\right)$

$= 2 - \frac{8}{3} - \frac{3}{2} + \frac{1}{3} + 2$

$= 4 - \frac{7}{3} - \frac{3}{2}$

$= 4 - 2\frac{1}{3} - 1\frac{1}{2}$

$= 1\frac{2}{3} - 1\frac{1}{2}$

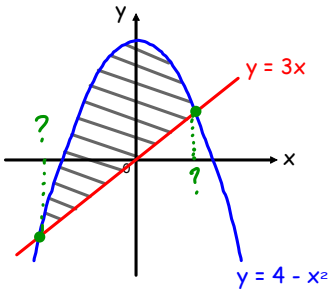
$= 1\frac{4}{6} - 1\frac{3}{6}$

$= \frac{1}{6}$ sq units.

- use simultaneous equations to find the points of intersection. (only require the x-cordinates)

2) Calculate the area enclosed(trapped) between the curve $y = 4 - x^2$ and the line $y = 3x$.

Working



POI

$y = 3x \quad y = 4 - x^2$

$3x = 4 - x^2$

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

$x = -4 \quad x = 1$

$$A = \int_{-4}^{1} (4 - x^2 - 3x)\,dx$$

$$= \left[4x - \frac{x^3}{3} - \frac{3x^2}{2}\right]_{-4}^{1}$$

$$= 4 \times 1 - \frac{1^3}{3} - \frac{3 \times 1^2}{2} - \left(4 \times (-4) - \frac{(-4)^3}{3} - \frac{3 \times (-4)^2}{2}\right)$$

$$= 4 - \frac{1}{3} - \frac{3}{2} - \left(-16 + \frac{64}{3} - 24\right)$$

$$= 4 - \frac{1}{3} - \frac{3}{2} + 16 - \frac{64}{3} + 24$$

$$= 44 - \frac{65}{3} - \frac{3}{2}$$

$$= 44 - 21\frac{2}{3} - 1\frac{1}{2}$$

$$= 22\frac{1}{3} - 1\frac{1}{2}$$

$$= 21\frac{2}{6} - \frac{3}{6}$$

$$= 21 - \frac{1}{6}$$

$20\frac{6}{6} - \frac{1}{6}$

$= 20\frac{5}{6}$

$= 20\frac{5}{6}$ sq units

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p180 Ex 9P Q1(a,b),4 & 5