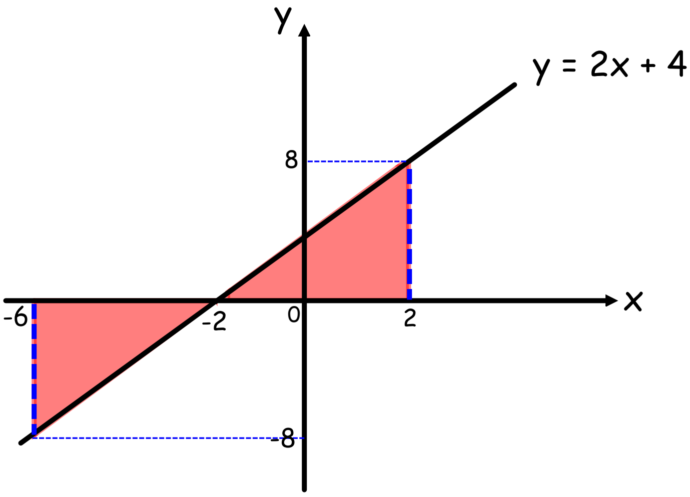
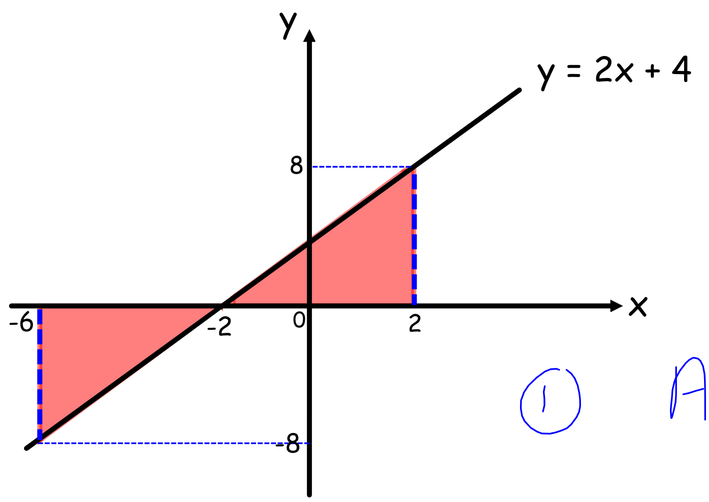
Calculating this Shaded Area



Can you calculate this Shaded Area?



Using Triangles

$A = 16 \times 2 = 32$ sq units

Using Integration

① $A = \int_{-6}^{2} (2x+4)\,dx = 0$ sq units

② $A = \int_{-2}^{2} (2x+4)\,dx$

$= \left[x^2 + 4x\right]_{-2}^{2}$

$= (4+8) - (4-8)$

$= 12 - (-4)$

$= 16$ sq units

$A = \int_{-6}^{-2} (2x+4)\,dx$

$= \left[x^2 + 4x\right]_{-6}^{-2}$

$= (4-8) - (36-24)$

$= -4 - 12$

$= -16$

So $A = 16$ sq units

Areas above & below the x-axis

When calculating areas by integration

- areas **above** the x-axis are **positive**
- areas **below** the x-axis are **negative**.

When calculating areas you need to

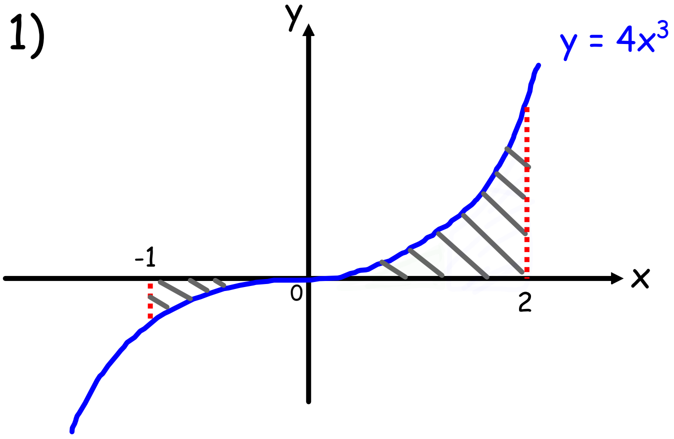
- calculate areas above and below the x-axis **separately**

unless you can use symmetry (line or rotational)

Examples

Calculate the **total** shaded area in each graph.

1)



Area above x-axis

$$A = \int_0^2 4x^3\,dx$$

$$= \left[x^4\right]_0^2$$

$$= 2^4 - 0^4$$

$= 16$ sq units.

Area below x-axis

$$A = \int_{-1}^0 4x^3\,dx$$

$$= \left[x^4\right]_{-1}^0$$

$$= 0^4 - (-1)^4$$

$$= 0 - 1$$

$$= -1$$

So $A = 1$ sq unit.

Total Area

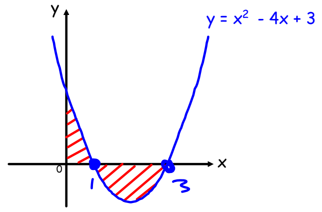
$= 16 + 1$

$= 17$ sq units.

Areas above & below the x-axis

p177 Ex 9N Q1(a & c)

2)



Roots $x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

Area above the x-axis:

① $A = \int_0^1 (x^2 - 4x + 3)\,dx$

$= \left[\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right]_0^1$

$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1$

$= \frac{1^3}{3} - 2\times1^2 + 3\times1 - \left(\frac{0^3}{3} - 2\times0^2 + 3\times0\right)$

$= \frac{1}{3} - 2 + 3 - 0$

$= 1\frac{1}{3}$ or $\frac{4}{3}$ sq units

Area below the x-axis:

② $A = \int_1^3 (x^2 - 4x + 3)\,dx$

$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^3$

$= \frac{3^3}{3} - 2\times3^2 + 3\times3 - \left(\frac{1^3}{3} - 2\times1^2 + 3\times1\right)$

$= 9 - 18 + 9 - \frac{4}{3}$

$= -\frac{4}{3}$ sq units

So $A = \frac{4}{3}$ sq units

Total Area:

Total $= \frac{4}{3} + \frac{4}{3}$

$= \frac{8}{3}$ sq units

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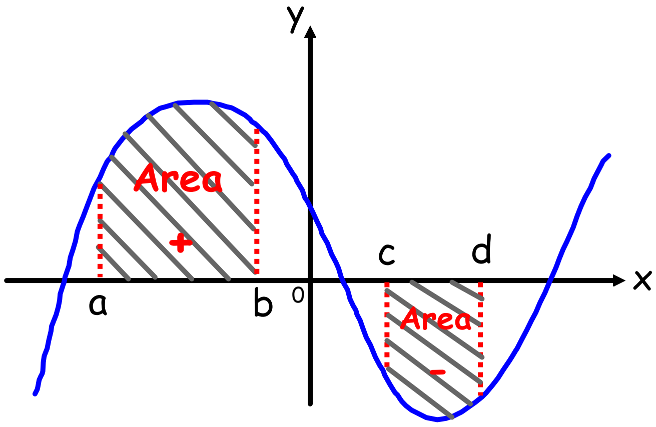

Find (i) $\int_a^b f(x)\ dx$ (ii) $\int_c^d f(x)\ dx$

(this will be negative)

then ignore negative sign and ADD the areas