

**What does the value of a Definite Integral tell us?**

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$$\int_1^7 (2x + 5) \, dx$$

$$= \mathbf{78}$$

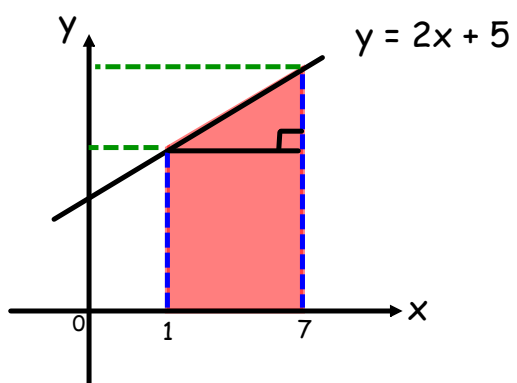
Find the value of this Integral.

$$\int_1^7 (2x + 5) \, dx$$

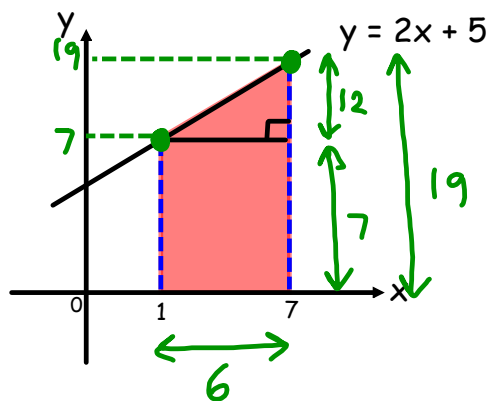
Find the value of this Integral.

$$\begin{aligned}
 & \int_1^7 (2x + 5) \, dx \\
 &= \left[ \frac{2x^2}{2} + 5x \right]_1^7 \\
 &= \left[ x^2 + 5x \right]_1^7 \\
 &= 7^2 + 5 \times 7 - (1^2 + 5 \times 1) \\
 &= 49 + 35 - (1 + 5) \\
 &= 49 + 35 - 1 - 5 \\
 &= 78
 \end{aligned}$$

Can you find the shaded area under this graph?



Can you find the shaded area **under** this graph?



$$\begin{aligned} \text{When } x = 1 \quad y &= 2x + 5 \\ &= 2 \times 1 + 5 \\ &= \underline{\underline{7}} \end{aligned}$$

$$\begin{aligned} x = 7 \quad y &= 2x + 5 \\ &= 2 \times 7 + 5 \\ &= \underline{\underline{19}} \end{aligned}$$

Rectangle

$$\begin{aligned} A &= 6 \times 7 \\ &= 42 \text{ sq units} \end{aligned}$$

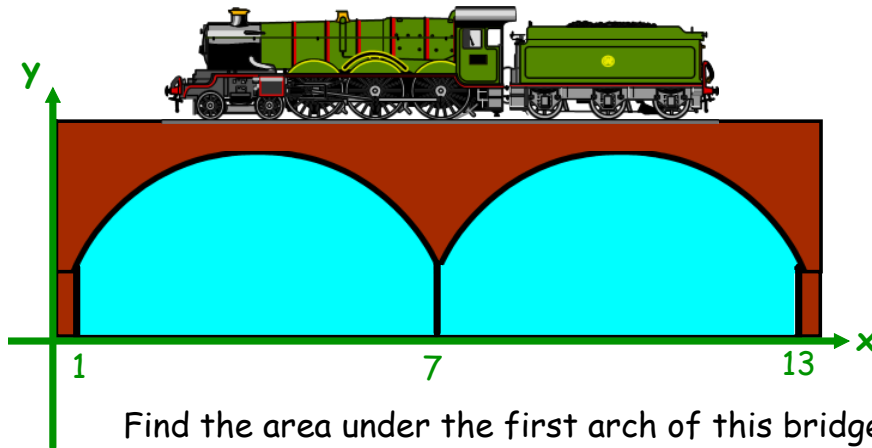
Triangle

$$\begin{aligned} A &= 6 \times 12 \div 2 \\ &= 36 \text{ sq units} \end{aligned}$$

$$\begin{aligned} \text{Total Area} &= 42 + 36 \\ &= 78 \text{ sq units} \end{aligned}$$

Notice  $\int_1^7 (2x + 5) dx = 78 \quad !!$

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Find the area under the first arch of this bridge.  
(The curve has equation  $y = 8x - x^2$ .)

$$A = \int_1^7 (8x - x^2) dx$$

$$= \left[ \frac{8x^2}{2} - \frac{x^3}{3} \right]_1^7$$

$$= \left[ 4x^2 - \frac{x^3}{3} \right]_1^7$$

$$= 4 \times 7^2 - \frac{7^3}{3} - \left( 4 \times 1^2 - \frac{1^3}{3} \right)$$

$$\begin{array}{r} 49 \\ \times 4 \\ \hline 196 \\ 3 \end{array}$$

$$= 196 - \frac{147}{3} - \left( 4 - \frac{1}{3} \right)$$

$$\begin{array}{r} 49 \\ \times 3 \\ \hline 147 \\ 2 \end{array}$$

$$= 196 - \frac{147}{3} - 4 - \frac{1}{3}$$

$$= 192 - \frac{148}{3}$$

$$\begin{array}{r} 192 \\ - 47 \\ \hline 145 \end{array}$$

$$= 192 - 47$$

$$= 145 \text{ sq units}$$

## Definite Integrals and Areas

The value of a definite integral

$$\int_a^b f(x) dx$$

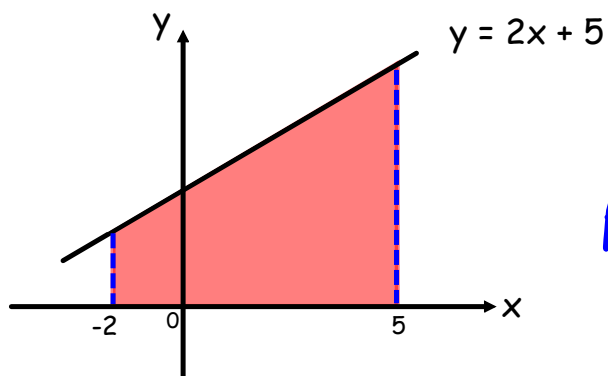
represents the area under the graph of  $y = f(x)$  and the  $x$ -axis between the vertical lines with equations  $x = a$  &  $x = b$ .



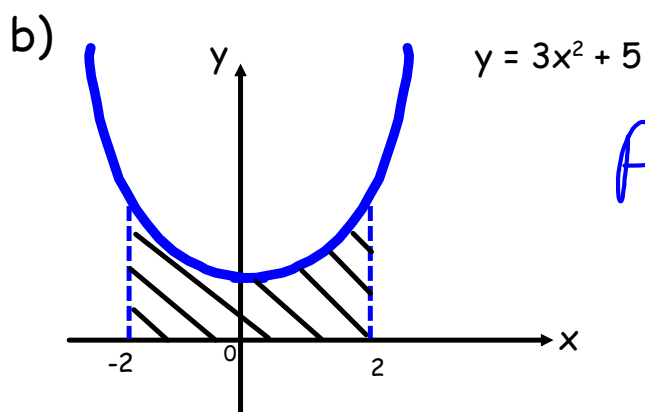
## Examples

1) Write the shaded area as a definite integral.

a)

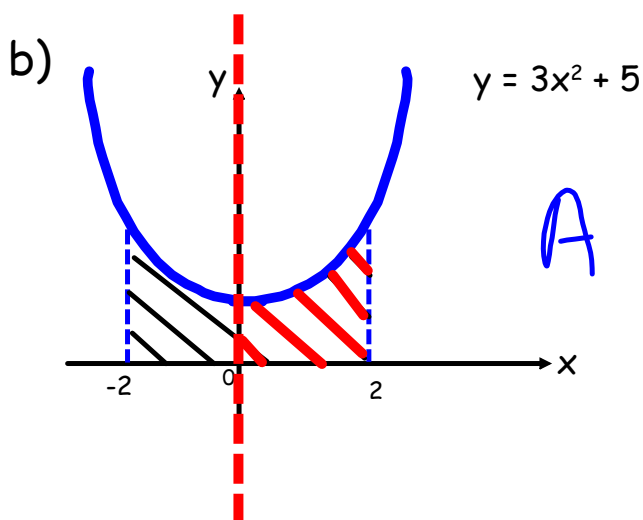


$$A = \int_{-2}^5 (2x + 5) dx$$

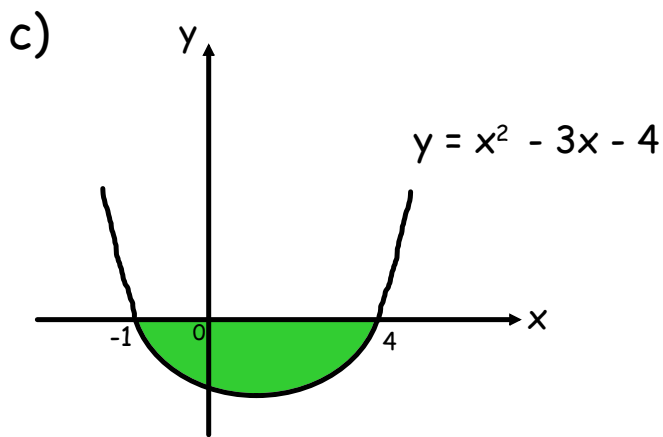


$$A = \int_{-2}^2 (3x^2 + 5) dx$$

OR  $\equiv$  by using symmetry

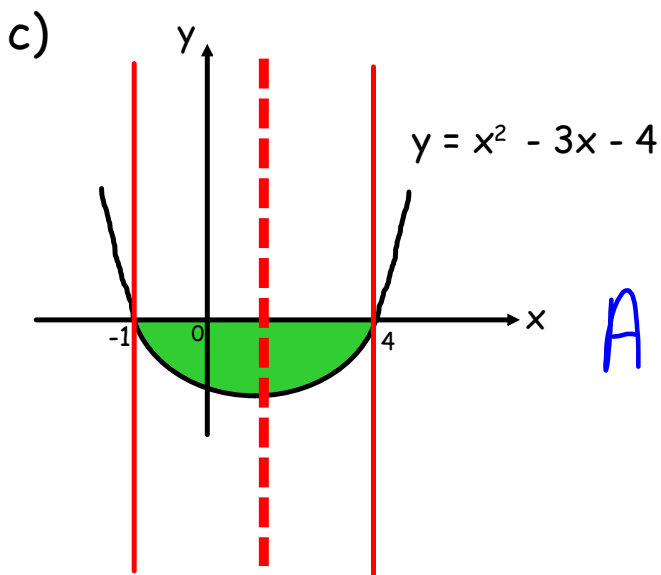


$$A = 2 \int_0^2 (3x^2 + 5) dx$$



$$A = \int_{-1}^4 (x^2 - 3x - 4) dx$$

OR  $\equiv$  by using symmetry



$$A = 2 \int_{\frac{3}{2}}^4 (x^2 - 3x - 4) dx$$

$x = -1$     $x = 4$

$$x = \frac{-1+4}{2}$$

$$x = \frac{3}{2}$$

## Definite Integrals and Areas

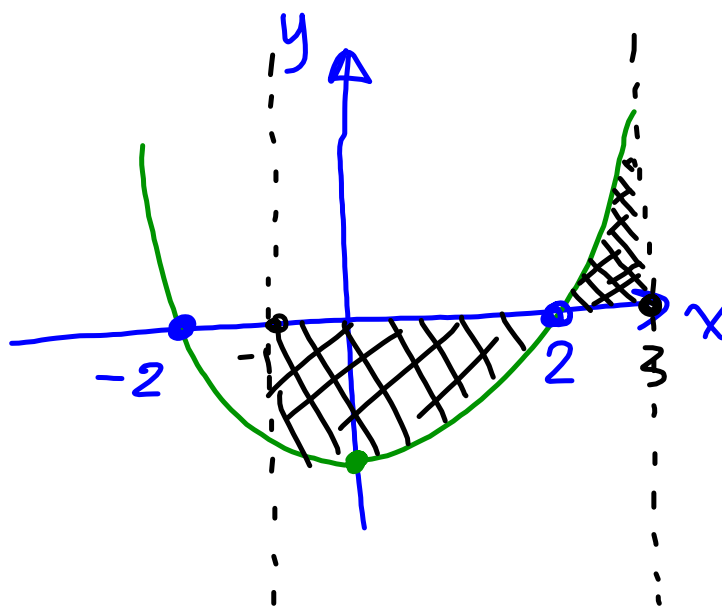
p173 Ex 9K Q1

2) Sketch the area represented by the definite Integral.

a)

$$\int_{-1}^3 (4x^2 - 16) dx$$

Working



Roots

$$4x^2 - 16 = 0$$

$$4x^2 = 16$$

$$x^2 = 4$$

$$x = \pm 2$$

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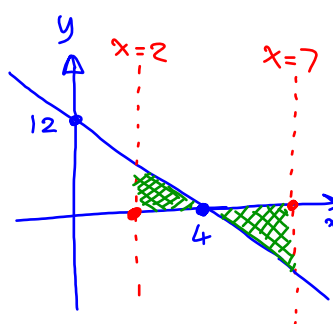
$$\text{b) } \int_2^7 (12 - 3x) \, dx$$

Working

$$y = 12 - 3x$$

Root (x-intercept)

$$\begin{aligned} 12 - 3x &= 0 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$



\*

p173 Ex 9K Q2