Definite Integrals

Definite Integrals

$$\boxed{\int_a^b f(x)\ dx}$$

This is called a definite integral.

'a' & 'b' are called the **<u>lower & upper limits</u>** of Integration respectively.

Evaluating Definite Integrals

Examples

Evaluate

1)

$$\int_1^4 2x^2 \quad dx$$

$$= \left[\frac{2x^3}{3}\right]_1^4$$

$$= \frac{2}{3} \times 4^3 - \frac{2}{3} \times 1^3$$

$$= \frac{2}{3} \times 64 - \frac{2}{3}$$

$$= \frac{128}{3} - \frac{2}{3}$$

$$= \frac{126}{3}$$

$$= 42$$

2) $$\int_{-2}^{3} (x^2 + 2x) \; dx$$

$$= \left[\frac{1}{3}x^3 + x^2\right]_{-2}^{3}$$

$$= \frac{1}{3} \times 3^3 + 3^2 - \left(\frac{1}{3} \times (-2)^3 + (-2)^2\right)$$

$$= 9 + 9 - \left(-\frac{8}{3} + 4\right)$$

$$= 18 + \frac{8}{3} - 4$$

$$= 14 + 2\frac{2}{3}$$

$$= \underline{\underline{16\frac{2}{3}}}$$

Evaluating Definite Integrals

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Q1(do every 2nd part)

e) $\int_2^5 (x^2 + 3x)\,dx$

$= \left[\frac{1}{3}x^3 + \frac{3}{2}x^2\right]_2^5$

$= \frac{1}{3} \times 5^3 + \frac{3}{2} \times 5^2 - \left(\frac{1}{3} \times 2^3 + \frac{3}{2} \times 2^2\right)$

$= \frac{125}{3} + \frac{75}{2} - \left(\frac{8}{3} + \frac{12}{2}\right)$

$= \frac{125}{3} - \frac{8}{3} + \frac{75}{2} - \frac{12}{2}$

$= \frac{117}{3} + \frac{63}{2}$

$= 39 + 31\frac{1}{2}$

$= \underline{\underline{70\frac{1}{2}}}$

3) $\int_{8}^{27} x^{\frac{1}{3}}\, dx$

$$\int_{8}^{27} x^{\frac{1}{3}}\, dx$$

$$= \left[\frac{3}{4} x^{\frac{4}{3}} \right]_{8}^{27}$$

When evaluating, you **MUST** first

- get **RID** of negative & fractional indices

before you substitute.

$$= \left[\frac{3}{4} \sqrt[3]{x}^{\,4} \right]_{8}^{27}$$

$$= \frac{3}{4} \sqrt[3]{27}^{\,4} - \frac{3}{4} \sqrt[3]{8}^{\,4}$$

$$= \frac{3}{4} \times 81 - \frac{3}{4} \times 16$$

$$= \frac{243}{4} - 12$$

$$\begin{array}{r} 81 \\ \times\ 3 \\ \hline 243 \end{array}$$

$$= 60\tfrac{3}{4} - 12$$

$$= \underline{\underline{48\tfrac{3}{4}}}$$

4) $\int_1^4 (3y^{\frac{3}{2}} - 5y^{-2})\ dy$

$$\int_1^4 \left(3y^{\frac{3}{2}} - 5y^{-2}\right) dy$$

$$= \left[\frac{2}{5} \times 3y^{\frac{5}{2}} - \frac{5y^{-1}}{-1}\right]_1^4$$

When evaluating, you **MUST** first
• get **RID** of negative & fractional indices
before you substitute.

$$= \left[\frac{6}{5}\sqrt{y}^{\,5} + \frac{5}{y}\right]_1^4$$

$$= \frac{6}{5} \times \sqrt{4}^{\,5} + \frac{5}{4} - \left(\frac{6}{5}\sqrt{1}^{\,5} + \frac{5}{1}\right)$$

$$= \frac{6}{5} \times 32 + \frac{5}{4} - \left(\frac{6}{5} + 5\right)$$

$$= \frac{192}{5} + \frac{5}{4} - \frac{6}{5} - 5$$

$$\begin{array}{r} 32 \\ \times\ 6 \\ \hline 192 \\ \hline 1 \end{array}$$

$$= 38\tfrac{2}{5} + 1\tfrac{1}{4} - 1\tfrac{1}{5} - 5$$

$$= 37\tfrac{1}{5} - 5 + 1\tfrac{1}{4}$$

$$= 32\tfrac{1}{5} + 1\tfrac{1}{4}$$

$$= 33\tfrac{1}{5} + \tfrac{1}{4}$$

$$= 33\tfrac{4}{20} + \tfrac{5}{20}$$

$$= 33\tfrac{9}{20}$$

Evaluating Definite Integrals

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Q2(b,c,f,h,i & l)

2) b) $\int_4^9 x^{\frac{1}{2}}\, dx$

$$= \left[\frac{2}{3} x^{\frac{3}{2}}\right]_4^9$$

$$= \left[\frac{2}{3} \sqrt{x^3}\right]_4^9$$

$$= \frac{2}{3}\sqrt{9}^3 - \frac{2}{3}\sqrt{4}^3$$

$$= \frac{2}{3} \times 27 - \frac{2}{3} \times 8$$

$$= 18 - \frac{16}{3}$$

$$= 18 - 5\frac{1}{3}$$

$$= \underline{\underline{12\frac{2}{3}}}$$

j) $\int_1^2 (2x^{-2} - x^{-3})\,dx$

$= \left[\frac{2x^{-1}}{-1} - \frac{x^{-2}}{-2} \right]_1^2$

$= \left[-2x^{-1} + \frac{1}{2}x^{-2} \right]_1^2$

$= \left[\frac{-2}{x} + \frac{1}{2x^2} \right]_1^2$

$= \frac{-2}{2} + \frac{1}{2\times 2^2} - \left(\frac{-2}{1} + \frac{1}{2\times 1^2} \right)$

$= -1 + \frac{1}{8} - \left(-2 + \frac{1}{2} \right)$

$= -1 + \frac{1}{8} + 2 - \frac{1}{2}$

$= 1 + \frac{1}{8} - \frac{4}{8}$

$= 1 - \frac{3}{8}$

$= \frac{5}{8}$

5) $\int_1^3 \frac{5}{x^2}\,dx$

$$\int_1^3 \frac{5}{x^2}\,dx$$

$$= \int_1^3 5x^{-2}\,dx$$

When evaluating, you **MUST** first

• get **RID** of negative & fractional indices

before you substitute.

$$= \left[\frac{5x^{-1}}{-1}\right]_1^3$$

$$= \left[-5x^{-1}\right]_1^3$$

$$= \left[-\frac{5}{x}\right]_1^3$$

$$= -\frac{5}{3} - \left(-\frac{5}{1}\right)$$

$$= -\frac{5}{3} + 5$$

$$= 5 - 1\tfrac{2}{3}$$

$$= 3\tfrac{1}{3}$$

6) $\int_1^{25} \frac{y^2 - \sqrt{y}}{y}\, dy$

$$\int_1^{25} \frac{y^2 - \sqrt{y}}{y}\, dy$$

$$= \int_1^{25} \frac{y^2 - y^{\frac{1}{2}}}{y}\, dy$$

$$= \int_1^{25} \left(\frac{y^2}{y} - \frac{y^{\frac{1}{2}}}{y}\right) dy$$

$$= \int_1^{25} \left(y - y^{-\frac{1}{2}}\right) dy$$

$$= \left[\frac{y^2}{2} - 2y^{\frac{1}{2}}\right]_1^{25}$$

When evaluating, you **MUST** first

- get **RID** of negative & fractional indices

before you substitute.

$$= \left[\frac{y^2}{2} - 2\sqrt{y}\right]_1^{25}$$

4/5 ✓

$$= \frac{25^2}{2} - 2\sqrt{25} - \left(\frac{1^2}{2} - 2\sqrt{1}\right)$$

$$= \frac{625}{2} - 10 - \left(\frac{1}{2} - 2\right)$$

25×25

$25 \times 20 = 500$

$25 \times 5 = 125$

$\underline{\underline{625}}$

$$= \frac{625}{2} - 10 - \frac{1}{2} + 2$$

$$= \frac{624}{2} - 8$$

$$= 312 - 8$$

$$= \underline{\underline{304}}$$

Evaluating Definite Integrals

p175 Ex 9L

Q3 (a,c,e,h,i,j & k)

k) $\int_1^4 (\sqrt{t} - 2)^2 \, dt$

$= \int_1^4 (t^{\frac{1}{2}} - 2)(t^{\frac{1}{2}} - 2) \, dt$

$= \int_1^4 t - 2t^{\frac{1}{2}} - 2t^{\frac{1}{2}} + 4 \, dt$

$= \int_1^4 (t - 4t^{\frac{1}{2}} + 4) \, dt$

$= \left[\frac{1}{2}t^2 - \frac{2}{3} \times 4t^{\frac{3}{2}} + 4t\right]_1^4$

$= \left[\frac{1}{2}t^2 - \frac{8}{3}\sqrt{t^3} + 4t\right]_1^4$

$= \frac{1}{2} \times 4^2 - \frac{8}{3} \times \sqrt{4^3} + 4 \times 4$
$- \left(\frac{1}{2} \times 1^2 - \frac{8}{3} \times \sqrt{1^3} + 4 \times 1\right)$

$= 8 - \frac{8}{3} \times 8 + 16)$
$- \left(\frac{1}{2} - \frac{8}{3} + 4\right)$

$= 8 - \frac{64}{3} + 16 - \frac{1}{2} + \frac{8}{3} - 4$

$= 20 - \frac{56}{3} - \frac{1}{2}$

$= 20 - 18\frac{2}{3} - \frac{1}{2}$

$= 1\frac{1}{3} - \frac{1}{2}$

$= 1\frac{2}{6} - \frac{3}{6}$

$= \frac{5}{6}$

$1\frac{2}{6} - \frac{3}{6}$

$= 1 - \frac{1}{6}$

$= \frac{6}{6} - \frac{1}{6} = \frac{5}{6}$

7) Find the positive value of 'a', for which:

$$\int_a^{2a} (1 + 2x)\ dx = 24$$

Working

$$\int_a^{2a} (1+2x)\,dx = 24 \quad \text{(equation)}$$

$$\left[x + x^2\right]_a^{2a} = 24$$

$$2a + (2a)^2 - (a + a^2) = 24$$

$$2a + 4a^2 - a - a^2 = 24$$

$$a + 3\underline{\underline{a^2}} = 24 \quad \text{(Quadratic equation)}$$

$$3a^2 + 1a - 24 = 0$$

$$(3a - 8)(a + 3) = 0$$

$$3a - 8 = 0 \quad \text{or} \quad a + 3 = 0$$

$$3a = 8 \qquad \cancel{a = -3}$$

$$\underline{\underline{a = \frac{8}{3}}}$$

p170 Ex 9L

Q4(b,d & f)

f)

$$\int_{-a}^{3a} (4x+7)\,dx = 120$$

$$\left[2x^2+7x\right]_{-a}^{3a} = 120$$

$$2\times(3a)^2+7\times 3a - \left(2\times(-a)^2+7\times(-a)\right) = 120$$

$$18a^2+21a-(2a^2-7a) = 120$$

$$18a^2+21a-2a^2+7a = 120$$

$$16a^2+28a-120 = 0$$

$$4a^2+7a-30 = 0$$

$$(4a+15)(a-2) = 0$$

$$4a+15=0$$

$$4a=-15$$

$$\cancel{a=\frac{-15}{4}}$$

$$a = 2$$