Further Integrals

Preparing before integrating

- Before integrating, you **must** rewrite the integral with x on the numerator **and** replace all roots.

Examples

Find the following integrals. (ie integrate)

1) $$\int \frac{6}{x^3}\,dx$$

$$= \int 6x^{-3}\,dx$$

$$= \frac{6x^{-2}}{-2} + C$$

$$= -3x^{-2} + C$$

$$= \frac{-3}{x^2} + C$$

2) $\int \frac{dx}{x^4}$

$= \int \frac{1}{x^4} \, dx$

$= \int x^{-4} \, dx$

$= \frac{1x^{-3}}{-3} + C$

$= -\frac{1}{3x^3} + C$

3) $\int \sqrt[5]{x}\ dx$

$$= \int x^{\frac{1}{5}}\ dx$$

$$= \frac{x^{\frac{6}{5}}}{\frac{6}{5}} + C$$

$$= \frac{5}{6} x^{\frac{6}{5}} + C$$

$$= \frac{5}{6} \sqrt[5]{x^6} + C$$

OR 3) $\int \sqrt[5]{x}\ dx$

$= \int x^{\frac{1}{5}}\ dx$

$= \frac{5}{6} x^{\frac{6}{5}} + C$

$= \frac{5}{6} \sqrt[5]{x^6} + C$

4) $$\int \frac{5}{8\ x^3}\ dx$$

$$= \int \frac{5}{8} x^{-\frac{3}{4}}\ dx$$

$$= {}^{1}\cancel{4} \times \frac{5}{\cancel{8}_{2}} x^{\frac{1}{4}} + C$$

$$= \frac{5}{2} x^{\frac{1}{4}} + C$$

$$= \frac{5}{2} \sqrt[4]{x} + C$$

OR 4) $$\int \frac{5}{8\ x^3}\ dx$$

$$= \int \frac{5}{8} x^{-\frac{3}{4}}\ dx$$

$$= \cancel{4}^{1} \times \frac{5}{\cancel{8}_{2}} x^{\frac{1}{4}} + C$$

$$= \frac{5}{2} \sqrt[4]{x} + C$$

Further Integrals

p171 Ex 9I Q1(a to f)

• **Remove all brackets and then integrate each term**

5) $\int (3y - 4)^2 \, dy$

$$= \int (3y-4)(3y-4)\, dy$$

$$= \int (9y^2 - 24y + 16)\, dy$$

$$= \frac{9y^3}{3} - \frac{24y^2}{2} + 16y + C$$

$$= 3y^3 - 12y^2 + 16y + C$$

p171 Ex 9I Q1(g to j)

- **Before integrating split fraction into separate fractions and simplify each term.**

6) $$\int \frac{t^2 + 2}{\sqrt{t}} \; dt$$

$$= \int \frac{t^2 + 2}{t^{\frac{1}{2}}} \, dt$$

$$= \int \left(\frac{t^2}{t^{\frac{1}{2}}} + \frac{2}{t^{\frac{1}{2}}} \right) dt$$

$$= \int \left(t^{\frac{3}{2}} + 2t^{-\frac{1}{2}} \right) dt$$

$$= \frac{2}{5} t^{\frac{5}{2}} + 2 \times 2t^{\frac{1}{2}} + C$$

$$= \frac{2}{5} \sqrt{t^5} + 4\sqrt{t} + C$$

7) $$\int \frac{3m^4 - 2}{7\sqrt{m^5}}\, dm$$

$$= \int \frac{3m^4 - 2}{7m^{5/2}}\, dm$$

$$4 - \frac{5}{2}$$
$$= 4 - 2\tfrac{1}{2}$$
$$= 1\tfrac{1}{2}$$

$$= \int \left(\frac{3m^4}{7m^{5/2}} - \frac{2}{7m^{5/2}}\right) dm$$

$$= \int \left(\frac{3}{7}m^{\frac{3}{2}} - \underline{\frac{2}{7}m^{-\frac{5}{2}}}\right) dm$$

$$= \frac{2}{5} \times \frac{3}{7}m^{\frac{5}{2}} + \frac{2}{3} \times \frac{2}{7}m^{-\frac{3}{2}} + C$$

$$= \frac{6}{35}\sqrt{m^5} + \frac{4}{21\sqrt{m^3}} + C$$

8) $$\int \left(g^4 + \frac{1}{3g} \right)^2 \, dg$$

$$= \int \left(g^{\frac{4}{3}} + \frac{1}{3}g^{-1}\right)\left(g^{\frac{4}{3}} + \frac{1}{3}g^{-1}\right) dg$$

$$= \int \left(g^{\frac{8}{3}} + \frac{1}{3}g^{\frac{1}{3}} + \frac{1}{3}g^{\frac{1}{3}} + \frac{1}{9}g^{-2}\right) dg$$

$$= \int \left(g^{\frac{8}{3}} + \frac{2}{3}g^{\frac{1}{3}} + \frac{1}{9}g^{-2}\right) dg$$

$$= \frac{3}{11}g^{\frac{11}{3}} + \frac{3}{4} \times \frac{2}{3}g^{\frac{4}{3}} - \frac{1}{9}g^{-1} + C$$

$$= \frac{3}{11}\sqrt[3]{g^{11}} + \frac{1}{2}\sqrt[3]{g^{4}} - \frac{1}{9g} + C$$

p171 Ex 9I Q1(k to t)

n) $\displaystyle\int \frac{y^6 - 1}{y^{3/2}}\, dy$

$\displaystyle= \int \left(\frac{y^6}{y^{3/2}} - \frac{1}{y^{3/2}} \right) dy$

$6 - \frac{3}{2}$

$= \frac{12}{2} - \frac{3}{2}$

$= \frac{9}{2}$

$\displaystyle= \int \left(y^{9/2} - y^{-\frac{3}{2}} \right) dy$

$= \frac{2}{11} y^{\frac{11}{2}} + 2y^{-\frac{1}{2}} + C$

$= \frac{2}{11} y^{\frac{11}{2}} + \frac{2}{y^{1/2}} + C$