Functions

1)

$f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$.

(a) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(b) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in its simplest form. **3**

a) $p(x) = f(g(x))$

$= f\left(\frac{3}{x}\right)$ ✓

$= 3 - \frac{3}{x}$ ✓

OR $= \frac{3x}{x} - \frac{3}{x}$

$= \frac{3x - 3}{x}$

b) $p(q(x))$

$= p\left(\frac{3}{3-x}\right)$ ✓ $p(x) = 3 - \frac{3}{x}$

$= 3 - \frac{3}{\frac{3}{3-x}}$ ✓

$= 3 - 3 \div \frac{3}{3-x}$

$= 3 - \cancel{3} \times \frac{3-x}{\cancel{3}}$

$= 3 - (3 - x)$

$= 3 - 3 + x$

$= x$ ✓

Functions

2)

Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

(*a*) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**

(*b*) Write down any restriction on the domain of h. **1**

a) $h(x) = f(g(x))$

$= f(2x+3)$ ✓

$= \dfrac{1}{2x+3-4}$

$= \dfrac{1}{2x-1}$ ✓

b) $2x-1 \neq 0$

$2x \neq 1$

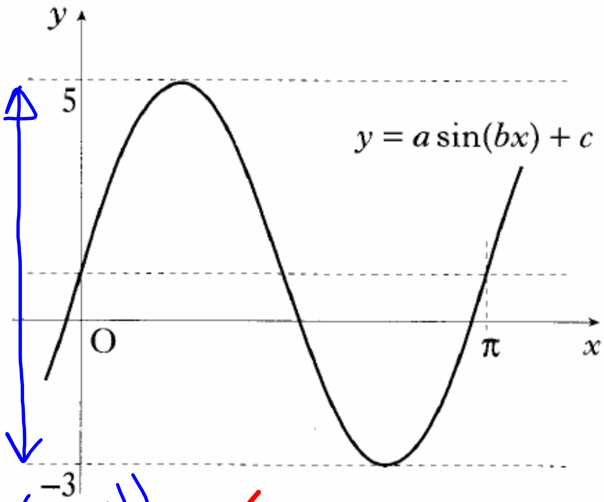
$x \neq \frac{1}{2}$ ✓

Functions

3)

The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.

Determine the values of a, b and c. 3



$a = 8 \div 2 = 4$ ✓

$b = 2$ (2 waves in 2π ($360°$)) ✓

$c = 1$ ($y = 4\sin 2x$ moved up by 1) ✓

Functions

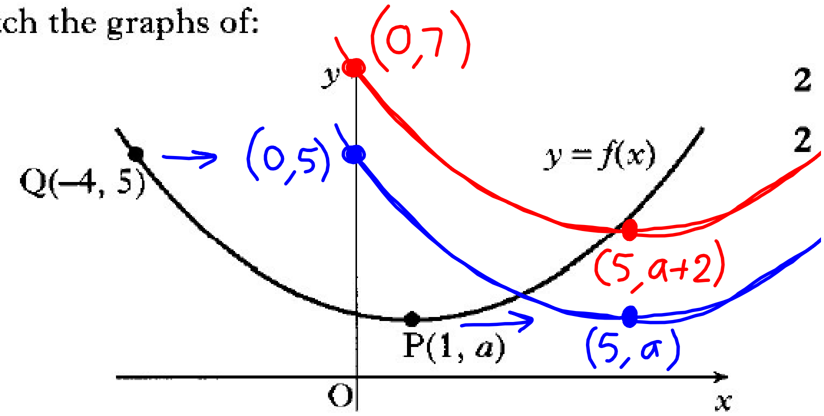
4)

The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of:

(*a*) $y = f(x - 4)$; **2**

(*b*) $y = 2 + f(x - 4)$. **2**



a) $y = f(x-4)$

is $y = f(x)$ moved 4 units to the right

b) $y = 2 + f(x-4)$

$= f(x-4)$ is graph of $y = (x-4)$ moved up 2units

✓ moving curve 4 to the right

✓ correct coordinates

✓ moving curve by by 4

✓ correct coordinates

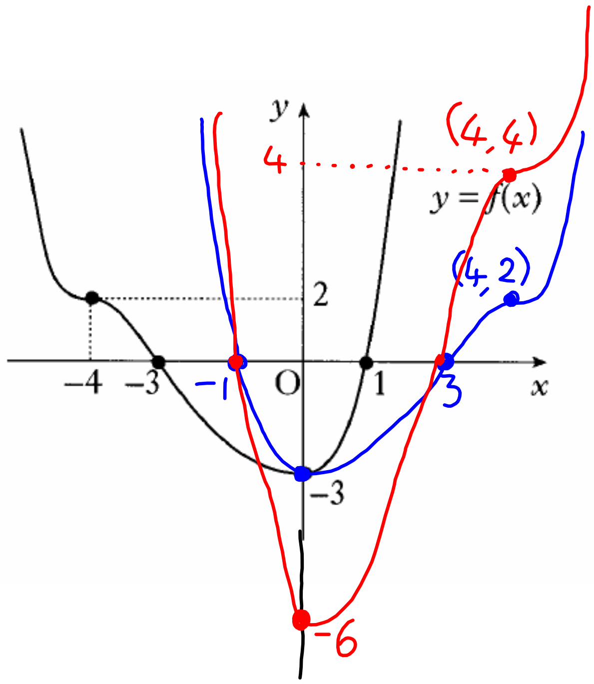
5)

The diagram shows the graph of a function f.

f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

(*a*) Sketch the graph of $y = f(-x)$. **2**

(*b*) On the same diagram, sketch the graph of $y = 2f(-x)$. **2**



a) $y = f(-x)$

is $y = f(x)$

reflected in y-axis

b) $y = 2f(-x)$

is $y = f(-x)$ from part (a)

stretched vertically. All

y coordinates x by 2.

✓ for moving graph correctly

✓ for correct coordinates

6)

The function f, defined on a suitable domain, is given by $f(x) = \frac{3}{x+1}$.

(*a*) Find an expression for $h(x)$ where $h(x) = f(f(x))$, giving your answer as a fraction in its simplest form. **3**

(*b*) Describe any restriction on the domain of h. **1**

a) $h(x) = f(f(x))$

$= f\left(\frac{3}{x+1}\right)$ ✓

$= \frac{3}{\frac{3}{x+1} + 1}$ ✓

$f(x) = \frac{3}{x+1}$

$= \frac{3}{\frac{3}{x+1} + \frac{x+1}{x+1}}$

$= \frac{3}{\frac{x+4}{x+1}}$

$= 3 \div \frac{x+4}{x+1}$

$= 3 \times \frac{x+1}{x+4}$

$= \frac{3(x+1)}{x+4}$ ✓

b) $x + 4 \neq 0$

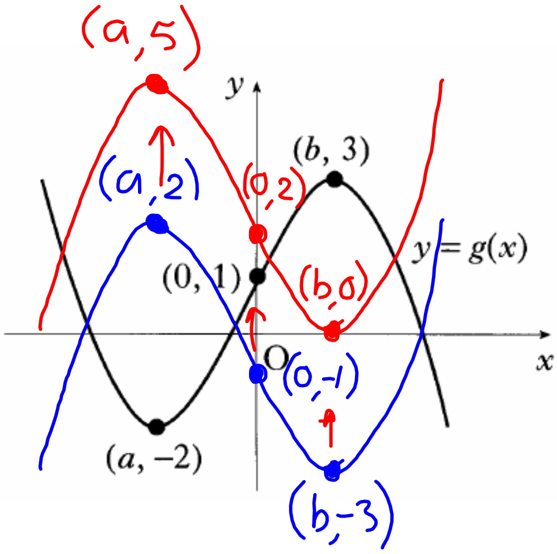
$x \neq -4$ ✓

7)

The diagram shows the graph of $y = g(x)$.

(*a*) Sketch the graph of $y = -g(x)$. **2**

(*b*) On the same diagram, sketch the graph of $y = 3 - g(x)$. **2**



a) $y = -g(x)$

is $y = g(x)$ reflected in x-axis

b) $y = 3 - g(x)$
$= -g(x) + 3$

is $y = -g(x)$ from (a) moved up by 3 units

✓ for moving graph correctly

✓ showing correct coordinates

8)

Functions f and g, defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find:

(*a*) $g(f(x))$; **2**

(*b*) $g(g(x))$. **2**

a) $g(f(x))$

$= g(x^2+1)$ ✓

$= 1 - 2(x^2+1)$

$= 1 - 2x^2 - 2$

$= -1 - 2x^2$ ✓

b) $g(g(x))$

$= g(1-2x)$ ✓

$= 1 - 2(1-2x)$

$= 1 - 2 + 4x$

$= 4x - 1$ ✓

9)

A function f is given by $f(x) = \sqrt{9 - x^2}$.

What is a suitable domain of f? **2**

$9 - x^2 \geq 0$ (quadratic inequality)

__Roots__ $9 - x^2 = 0$

$x^2 = 9$

$x = \pm 3$

Domain

$-3 \leq x \leq 3$

or $x \leq 3$

and $x \geq -3$