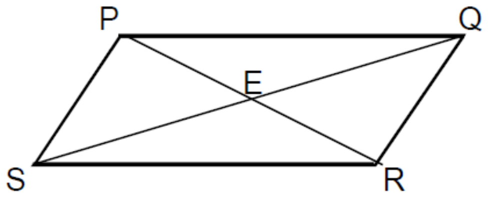
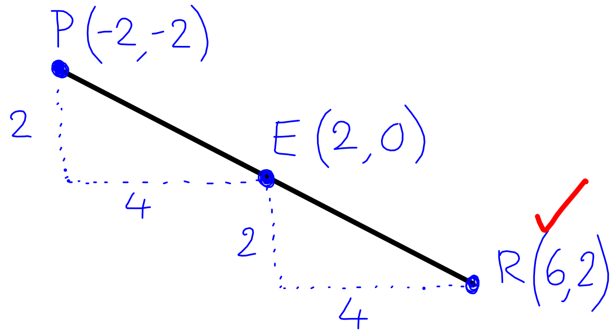
2. PQRS is a parallelogram whose diagonals meet at E.
P is the point (-2,-2), Q is (0,2) and E is (2,0).
Find the equation of the line RS.



$M_{PQ} = M_{RS}$



$M_{PQ} = \frac{2+2}{0+2}$

$= \frac{4}{2}$

$= 2$ ✓

So $M_{RS} = 2$

$R(\underset{a}{6}, \underset{b}{2})$

$y - b = m(x - a)$

$y - 2 = 2(x - 6)$ ✓

$y - 2 = 2x - 12$

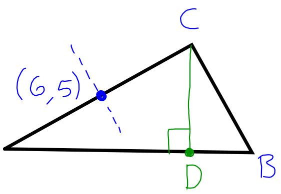
$2x - y - 10 = 0$ ✓

OR, $y = 2x - 10$

3. A triangle ABC has vertices A(2,5). B(4,-1) and C(10,5).

(a) Write down the equation of the perpendicular bisector of AC.
(b) Find the equation of the altitude CD.
(c) Find the point of intersection of these two lines.

a)



$$\text{Mid-pt of } AC = \left(\frac{2+10}{2}, \frac{5+5}{2}\right)$$

$$= (6, 5) \checkmark$$

$$m_{AC} = \frac{5-5}{10-2}$$

$$= \frac{0}{8}$$

$$= 0 \checkmark \text{ (Horizontal)}$$

$$m_{perp} = \infty \text{ (Vertical)} \checkmark$$

Equation $x = 6$ ✓

b) $$m_{AB} = \frac{5+1}{2-4}$$ $$A(\underset{x}{2}, \underset{y}{5}) \quad B(\underset{x}{4}, \underset{y}{-1})$$

$$= \frac{6}{-2}$$

$$= -3 \checkmark$$

So $m_{alt} = \frac{1}{3}$ ✓ $C(\underset{a}{10}, \underset{b}{5})$

$$y - b = m(x - a)$$

$$y - 5 = \tfrac{1}{3}(x - 10) \checkmark \quad (\times 3)$$

$$3y - 15 = x - 10$$

$$x - 3y + 5 = 0 \checkmark$$

c) $x = 6$

$$x - 3y + 5 = 0$$

$$6 - 3y + 5 = 0 \checkmark$$

$$-3y + 11 = 0$$

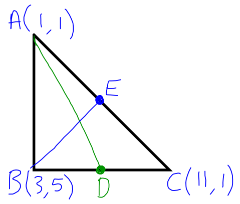
$$3y = 11$$

$$\left.\begin{array}{l} y = \frac{11}{3} \\ \left(6, \frac{11}{3}\right) \end{array}\right\} \checkmark$$

4. A triangle has vertices A(1,1), B(3,5) and C(11,1).

(a) Show that triangle ABC is right angled at B.
(b) Find the equations of the medians AD and BE.
(c) AD and BE intersect at M. Find the coordinates of M.

a)



$M_{AB} = \frac{5-1}{3-1}$

$= \frac{4}{2}$

$= 2$ ✓

$M_{BC} = \frac{5-1}{3-11}$

$= \frac{4}{-8}$

$= -\frac{1}{2}$ ✓

The triangle is right-angled at B because $M_{AB} \times M_{BC} = -1$. ✓

b) Mid-pt $BC = \left(\frac{3+11}{2}, \frac{5+1}{2}\right)$

$= (\underset{a}{7}, \underset{b}{3})$ ✓

$M_{med} = \frac{3-1}{7-1}$

(or M_{AD}) $= \frac{2}{6}$

$= \frac{1}{3}$ ✓

$y - b = m(x-a)$

$y - 3 = \frac{1}{3}(x-7)$ ✓ $\times 3$

$3y - 9 = x - 7$

$x - 3y + 2 = 0$ ✓

Median BE

Mid-pt $AC = \left(\frac{1+11}{2}, \frac{1+1}{2}\right)$

$= (\underset{a}{6}, \underset{b}{1})$ ✓

$M_{med} = \frac{5-1}{3-6}$

(or M_{BE}) $= \frac{4}{-3}$ ✓

$y - b = m(x-a)$

$y - 1 = -\frac{4}{3}(x-6)$ ✓ $\times 3$

$3y - 3 = -4(x-6)$

$3y - 3 = -4x + 24$

$4x + 3y - 27 = 0$ ✓

c)

$$\begin{array}{lrl} & x - 3y + 2 &= 0 \\ & 4x + 3y - 27 &= 0 \\ \hline \text{ADD} & 5x \quad - 25 &= 0 \end{array}$$

$5x = 25$

$x = 5$ ✓

find y

$x - 3y + 2 = 0$

$5 - 3y + 2 = 0$

$7 - 3y = 0$

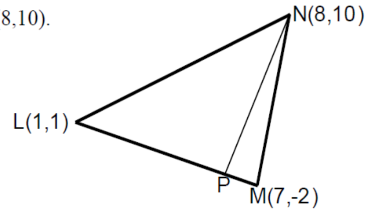
$7 = 3y$

$y = \frac{7}{3}$

$\left(5, \frac{7}{3}\right)$ ✓

5. A triangle has vertices L(1,1), M(7,-2) and N(8,10).

(a) Find the equation of the altitude NP.
(b) Find the coordinates of P.



a) $M_{LM} = \frac{1+2}{1-7}$

$= \frac{3}{-6}$

$= -\frac{1}{2}$ ✓

$M_{alt} = 2$ ✓ $(\overset{a}{8}, \overset{b}{10})$

$y - b = m(x - a)$

$y - 10 = 2(x - 8)$ ✓

$y - 10 = 2x - 16$

$2x - y - 6 = 0$ or $y = 2x - 6$ ✓

b) Require equation of line LM

$M_{LM} = -\frac{1}{2}$ (from (a)) $(\underset{a}{1}, \underset{b}{1})$

$y - b = m(x - a)$

$y - 1 = -\frac{1}{2}(x - 1)$ ✓ $\boxed{\times 2}$

$2y - 2 = -(x - 1)$

$2y - 2 = -x + 1$

$\underline{x + 2y - 3 = 0}$ ✓

$x + 2y - 3 = 0$

$2x - y - 6 = 0 \quad \times 2$

$x + 2y - 3 = 0$ ✓

$4x - 2y - 12 = 0$

ADD $5x \quad - 15 = 0$

$5x = 15$

$x = 3$ ✓

find y

$x + 2y - 3 = 0$

$3 + 2y - 3 = 0$

$2y = 0$

$y = 0$

$(3, 0)$ ✓

7. Triangle DEF has vertices (2,3), (-3,-2) and (3,0) respectively.

(a) Find the equations of the perpendicular bisectors of the sides EF and DF.
(b) Find the coordinates of T, the point of intersection of these lines.
(c) Show that D, T and E are collinear.

a) $M_{EF} = \frac{-2-0}{-3-3}$

$= \frac{-2}{-6}$

$= \frac{1}{3}$ ✓

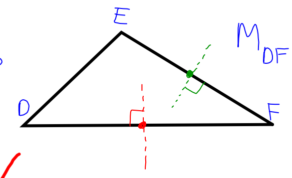
So $M_{perp} = -3$ ✓

Mid-pt $EF = \left(\frac{-3+3}{2}, \frac{-2+0}{2}\right)$

$= (0, -1)$ ✓

y-intercept so use $y = mx + c$

$y = -3x - 1$ ✓



$M_{DF} = \frac{3-0}{2-3}$

$= \frac{3}{-1}$

$= -3$ ✓

$M_{perp} = \frac{1}{3}$ ✓

Mid-pt $DF = \left(\frac{2+3}{2}, \frac{3+0}{2}\right)$

$= \left(\frac{5}{2}, \frac{3}{2}\right)$ ✓

$y - b = m(x - a)$

$y - \frac{3}{2} = \frac{1}{3}\left(x - \frac{5}{2}\right)$ $\boxed{\times \text{by } 6}$

$6y - 9 = 2\left(x - \frac{5}{2}\right)$

$6y - 9 = 2x - 5$

$2x - 6y + 4 = 0$ ✓ $(\div \text{ by } 2)$

$x - 3y + 2 = 0$

b) $x - 3y + 2 = 0$ $\quad y = -3x - 1$

$x - 3(-3x - 1) + 2 = 0$ ✓

$x + 9x + 3 + 2 = 0$

$10x + 5 = 0$

$10x = -5$

$x = \frac{-5}{10}$ ✓

$= -\frac{1}{2}$

find y

$y = -3x - 1$

$= -3 \times \left(-\frac{1}{2}\right) - 1$

$= \frac{3}{2} - \frac{2}{2}$

$= \frac{1}{2}$ ✓ $T\left(-\frac{1}{2}, \frac{1}{2}\right)$

c) $D(2,3)$ $\quad T\left(-\frac{1}{2}, \frac{1}{2}\right)$ $\quad E(-3,-2)$

$M_{DT} = \frac{\frac{1}{2} - 3}{-\frac{1}{2} - 2}$

$= \frac{-2.5}{-2.5}$

$= 1$ ✓

$M_{TE} = \frac{-2 - \frac{1}{2}}{-3 + \frac{1}{2}}$

$= \frac{-2.5}{-2.5}$

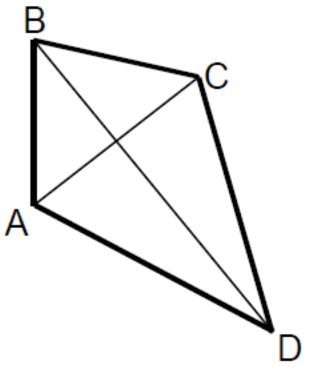
$= 1$ ✓

Lines DT and TE are parallel since $M_{DT} = M_{TE}$ and the points D, T and E are collinear since T is a common point. ✓

10. A kite ABCD has diagonals AC and BD.

AC has equation 2y = x – 2.
D is the point (6,-3).

(a) Find the equation of the diagonal BD.
(b) Find the coordinates of the point of intersection of these diagonals.



a) $2y = x - 2$ (÷ by 2)

$y = \frac{1}{2}x - 1$

$m = \frac{1}{2}$ ✓

$m_{perp} = -2$ ✓ $(\overset{}{\underset{a}{6}}, \underset{b}{-3})$

$y - b = m(x - a)$

$y + 3 = -2(x - 6)$

$y + 3 = -2x + 12$

$2x + y - 9 = 0$ ✓

or $y = -2x + 9$

b) $x - 2y - 2 = 0$

$2x + y - 9 = 0$ × 2

$x - 2y - 2 = 0$

$4x + 2y - 18 = 0$

ADD $5x - 20 = 0$

$5x = 20$ ✓

$x = 4$

find y

$x - 2y - 2 = 0$

$4 - 2y - 2 = 0$

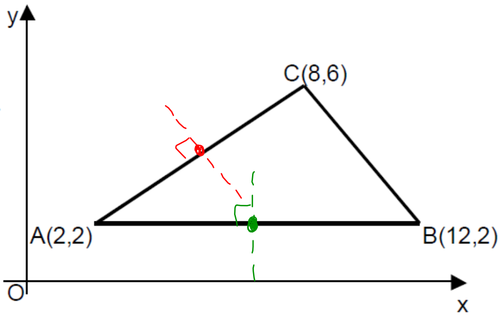
$2 - 2y = 0$

$2y = 2$

$y = 1$ (4, 1) ✓

11. Triangle ABC has vertices A(2,2), B(12,2) and C(8,6).

(a) Write down the equation of the perpendicular bisector of AB.
(b) Find the equaton of the perpendicular bisector of AC.
(c) Find the point of intersection of these lines.



A(2,2) B(12,2)

a) $M_{AB} = \frac{2-2}{12-2}$

$= \frac{0}{10}$

$= 0$ ✓ (Horizontal)

So $M_{perp} = \infty$ ✓ (Vertical)

Mid-pt AB $= \left(\frac{2+12}{2}, \frac{2+2}{2}\right)$

$= (7, 2)$ ✓

So $x = 7$ ✓

b) Mid-pt AC $= \left(\frac{2+8}{2}, \frac{2+8}{2}\right)$

$= (5, 4)$ ✓

$M_{AC} = \frac{6-2}{8-2}$

$= \frac{4}{6}$

$= \frac{2}{3}$ ✓

So $M_{perp} = -\frac{3}{2}$ ✓

$y - b = m(x - a)$ $\quad (\underset{a}{5}, \underset{b}{4})$

$y - 4 = -\frac{3}{2}(x - 5)$ $\boxed{\times 2}$

$2y - 8 = -3(x - 5)$

$2y - 8 = -3x + 15$ ✓

$3x + 2y - 23 = 0$

c) $x = 7$ from part (a)

$3x + 2y - 23 = 0$

$3 \times 7 + 2y - 23 = 0$ ✓

$21 + 2y - 23 = 0$

$2y - 2 = 0$

$2y = 2$

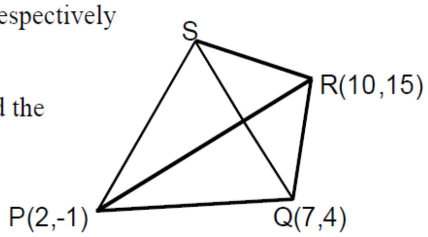
$y = 1$ ✓

$(7, 1)$

12. P, Q and R have coordinates (2,-1), (7,4) and (10,15) respectively and are three vertices of a kite PQRS.

(a) Find the equations of the diagonals of this kite and the coordinates of the point where they intersect.

(b) Find the coordinates of the fourth vertex S.



a). Line PR

$m_{PR} = \frac{15+1}{10-2}$

$= \frac{16}{8}$

$= 2$ ✓ $(\underset{a}{2}, \underset{b}{-1})$

$y - b = m(x - a)$

$y + 1 = 2(x - 2)$

$y + 1 = 2x - 4$

$2x - y - 5 = 0$ ✓

Line QS

$m_{QS} = -\frac{1}{2}$ ✓ (since $m_{PR} = 2$)

$(\underset{a}{7}, \underset{b}{4})$

$y - b = m(x - a)$

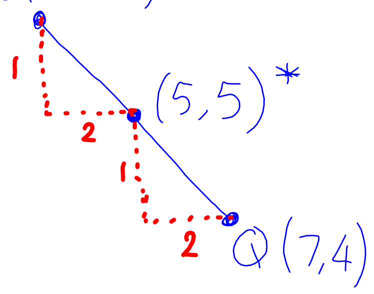
$y - 4 = -\frac{1}{2}(x - 7)$ $(\times 2)$

$2y - 8 = -(x - 7)$

$2y - 8 = -x + 7$ ✓

$x + 2y - 15 = 0$

b). S(   )



So S(5-2, 5+1)

S(3,6) ✓

(with diagram)

$* \quad 2x - y - 5 = 0 \quad (\times 2)$

$x + 2y - 15 = 0$

$4x - 2y - 10 = 0$

$x + 2y - 15 = 0$

$5x \quad - 25 = 0$

$5x = 25$ ✓

$x = 5$

find y $\quad 2x - y - 5 = 0$

$2 \times 5 - y - 5 = 0$

$10 - y - 5 = 0$

$5 - y = 0$ ✓

$y = 5$ (5,5)