Differentiation Non-Calculator HW

1)

A curve has equation $y = x - \frac{16}{\sqrt{x}}$, $\quad x > 0$.

Find the equation of the tangent at the point where $x = 4$. **6**

use $y - b = m(x - a)$

find y

$$y = x - \frac{16}{\sqrt{x}}$$

$$y = 4 - \frac{16}{\sqrt{4}}$$

$$= 4 - \frac{16}{2}$$

$$= 4 - 8$$

$$= -4 \checkmark$$

$$(\underset{a}{4}, \underset{b}{-4})$$

find m

$$y = x - \frac{16}{\sqrt{x}}$$

$$= x - 16x^{-\frac{1}{2}} \checkmark$$

$$\frac{dy}{dx} = 1 + 8x^{-\frac{3}{2}} \checkmark$$

$$= 1 + \frac{8}{\sqrt{x^3}}$$

$$m = 1 + \frac{8}{\sqrt{4^3}}$$

$$= 1 + \frac{8}{8}$$

$$= 2 \checkmark$$

$m = 2 \quad (\underset{a}{4}, \underset{b}{-4})$

$$y - b = m(x - a)$$

$$y + 4 = 2(x - 4) \checkmark$$

$$y + 4 = 2x - 8$$

$$y = 2x - 12 \checkmark$$

2)

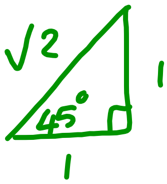
Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis. **4**

Need to find x coordinate first to allow you calculate the y-coordinate using $y = 2x^2 - 7x + 10$

$m = \tan\theta$

$m = \tan 45°$

$= 1$ ✓



also $m = \frac{dy}{dx}$

$\frac{dy}{dx} = 4x - 7$ ✓

So $4x - 7 = 1$

$4x = 8$

$x = 2$ ✓

find y

$y = 2x^2 - 7x + 10$

$y = 2 \times 2^2 - 7 \times 2 + 10$

$= 8 - 14 + 10$

$= 4$

So point is $(2, 4)$ ✓

3)

Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

$$f(x) = \sqrt{x} + \frac{2}{x^2}$$

$$= x^{\frac{1}{2}} + 2x^{-2}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 4x^{-3}$$

$$= \frac{1}{2\sqrt{x}} - \frac{4}{x^3}$$

$$f'(4) = \frac{1}{2\sqrt{4}} - \frac{4}{4^3}$$

$$= \frac{1}{4} - \frac{4}{64}$$

$$= \frac{1}{4} - \frac{1}{16}$$

$$= \frac{4}{16} - \frac{1}{16}$$

$$= \frac{3}{16}$$

4)

Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$. **4**

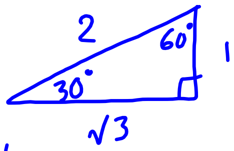
use $y - b = m(x - a)$

need y $\quad y = 2\sin\left(x - \frac{\pi}{6}\right)$

$= 2\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$ or $2\sin(60° - 30°)$

$= 2\sin\left(\frac{2\pi}{6} - \frac{\pi}{6}\right)$ $\quad = 2\sin 30°$

$= 2\sin\left(\frac{\pi}{6}\right)$



$= 2 \times \frac{1}{2}$

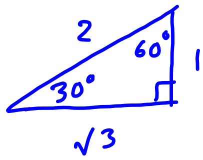
$= 1 \Rightarrow \left(\frac{\pi}{3}, 1\right)$ (a, b)

find m $\quad y = 2\sin\left(x - \frac{\pi}{6}\right)$

$\frac{dy}{dx} = 2\cos\left(x - \frac{\pi}{6}\right)$

$\left(x = \frac{\pi}{3}\right) \quad m = 2\cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$

$= 2\cos 30°$



$= 2 \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}$

$y - b = m(x - a) \quad m = \sqrt{3} \quad \left(\frac{\pi}{3}, 1\right)$

$y - 1 = \sqrt{3}\left(x - \frac{\pi}{3}\right)$

$y - 1 = \sqrt{3}x - \frac{\sqrt{3}\pi}{3}$

$y = \sqrt{3}x - \frac{\sqrt{3}\pi}{3} + 1$

5)

If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

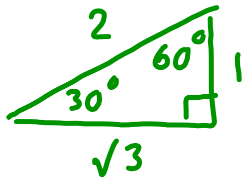
$$f(x) = \cos(2x) - 3\sin(4x)$$

$$f'(x) = -2\sin(2x) - 4 \times 3\cos(4x)$$

$$= -2\sin(2x) - 12\cos(4x)$$

$$f'\left(\frac{\pi}{6}\right) = -2\sin\left(\frac{2\pi}{6}\right) - 12\cos\left(\frac{4\pi}{6}\right)$$

$$= -2\sin 60^\circ - 12\cos 120^\circ$$



$$= -2 \times \frac{\sqrt{3}}{2} - 12 \times \left(-\frac{1}{2}\right)$$

A S T C

$180 - a$

$\cos 120^\circ$

$= -\cos 60^\circ$

$= -\frac{1}{2}$

$$= -\sqrt{3} + 6$$

$$= \underline{\underline{6 - \sqrt{3}}}$$

6)

The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

(*a*) Find the value of x for which the gradient of the tangent at P is 12. **5**

(*b*) Hence find the equation of the tangent at P. **2**

a) $m = 12$ So $\frac{dy}{dx} = 12$

$y = 6x^2 - x^3$

$\frac{dy}{dx} = 12x - 3x^2$ ✓ ✓

$12 = 12x - 3x^2$ ✓ (Quadratic Equation)

$3x^2 - 12x + 12 = 0$

$x^2 - 4x + 4 = 0$ ✓

$(x - 2)(x - 2) = 0$

$x = 2$ (twice) ✓

6)

The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

(a) Find the value of x for which the gradient of the tangent at P is 12. **5**

(b) Hence find the equation of the tangent at P. **2**

use $y - b = m(x - a)$ $m = 12$

find y ($x = 2$ from part (a))

$$y = 6x^2 - x^3$$
$$= 6 \times 2^2 - 2^3$$
$$= 24 - 8$$
$$= 16 \checkmark$$

$(\underset{a}{2}, \underset{b}{16})$ $m = 12$

$$y - b = m(x - a)$$
$$y - 16 = 12(x - 2)$$
$$y - 16 = 12x - 24$$
$$y = 12x - 8 \checkmark$$

7)

Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$. **3**

$$y = \sqrt{3x^2+2}$$

$$= (3x^2+2)^{\frac{1}{2}} \checkmark$$

(Use CHAIN RULE)

$$\frac{dy}{dx} = \frac{1}{2}(3x^2+2)^{-\frac{1}{2}} \times 6x$$

$$= 3x(3x^2+2)^{-\frac{1}{2}}$$

$$= \frac{3x}{\sqrt{3x^2+2}}$$

8)

A function f is defined by $f(x) = (2x - 1)^5$.
Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

So $f'(x) = 0$

$f(x) = (2x-1)^5$ (use CHAIN RULE to differentiate)

$f'(x) = 5(2x-1)^4 \times 2$

$= 10(2x-1)^4$

At S.P's $f'(x) = 0$

$10(2x-1)^4 = 0$

$(2x-1)^4 = 0$

$2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

find y $\quad y = f(x) = (2x-1)^5$

$= \left(2 \times \frac{1}{2} - 1\right)^5$

$= (1-1)^5$

$= 0 \quad \left(\frac{1}{2}, 0\right)$

Nature

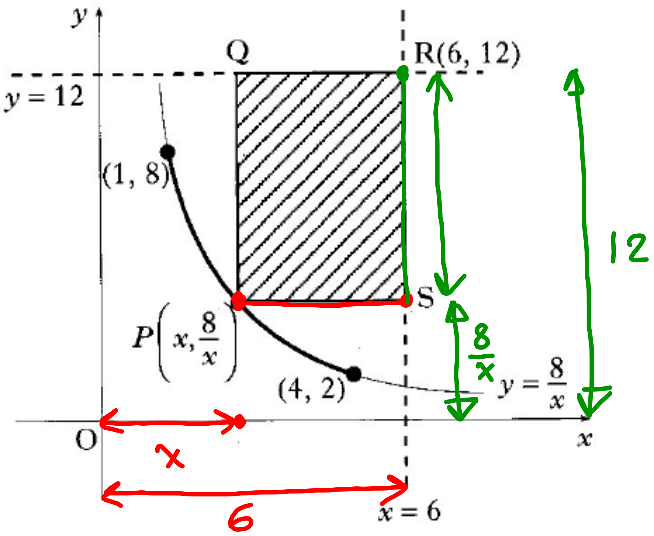
x	$0 \rightarrow$	$\frac{1}{2}$	$3 \rightarrow$
$m = f'(x) = 10(2x-5)^4$	$+$	0	$+$
SHAPE	/	—	/

(Rising) Point of inflection at $\left(\frac{1}{2}, 0\right)$

9)

PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(a) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by
$$A = 80 - 12x - \frac{48}{x}.$$ **3**

(b) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

(a) i) $PS = 6 - x$ $\quad RS = 12 - \frac{8}{x}$ ✓ (both)

(ii) $A = L \times B$

$A = (6 - x)\left(12 - \frac{8}{x}\right)$ ✓

$= 72 - \frac{48}{x} - 12x + 8$

$= 80 - 12x - \frac{48}{x}$ ✓

as required

9)

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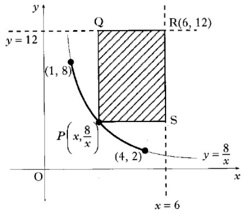

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b) $A = 80 - 12x - \frac{48}{x}$

$= 80 - 12x - 48x^{-1}$

$\frac{dA}{dx} = -12 + 48x^{-2}$

At S.P's $\frac{dA}{dx} = 0$

$-12 + 48x^{-2} = 0$

$\frac{48}{x^2} - 12 = 0$ (× by x^2 to clear fraction)

$48 - 12x^2 = 0$

$12x^2 = 48$

$x^2 = 4$

$x = 2$ or $\cancel{-2}$

since $1 \leq x \leq 4$

Nature

x	$\xrightarrow{1}$	2	$\xrightarrow{3}$
$M = \frac{dA}{dx} = \frac{48}{x^2} - 12$	+	0	−
SHAPE	/	—	\

Max T.P when $x = 2$

So Max Area when $x = 2$

$A = 80 - 12x - \frac{48}{x}$

$A = 80 - 12 \times 2 - \frac{48}{2}$

$= 80 - 24 - 24$

$= 32$ sq units.

Check for End-Points $(\underset{EP}{1} \leq x \leq \underset{EP}{4})$

$x = 1$ $A = 80 - 12x - \frac{48}{x}$

$= 80 - 12 \times 1 - \frac{48}{1}$

$= 80 - 12 - 48$

$= 20$ sq units

$x = 4$ $A = 80 - 12 \times 4 - \frac{48}{4}$

$= 80 - 48 - 12$

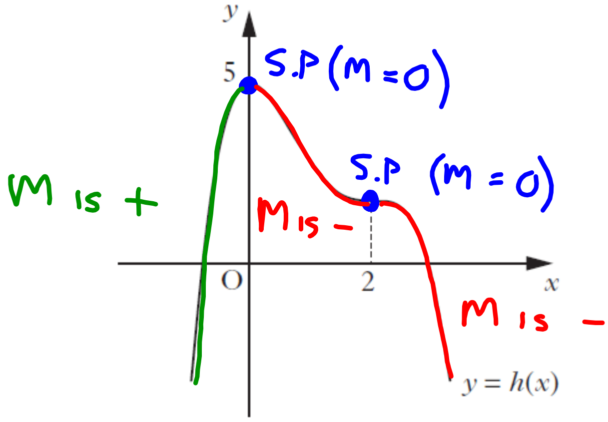
$= 20$ sq units

So min area $= 20$ sq units.

when $x = 1$ and $x = 4$

10)

The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.

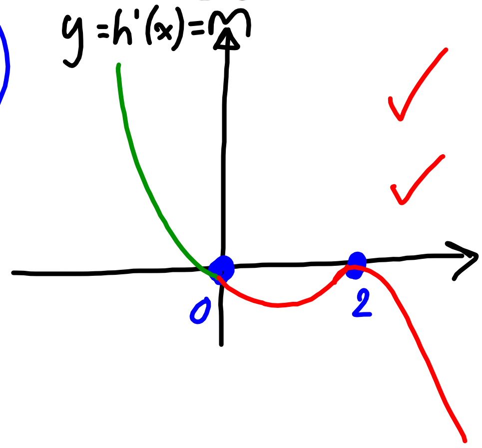


On separate diagrams sketch the graphs of:

(a) $y = h'(x)$; 3

(b) $y = 2 - h'(x)$. 3

a)



✓ (Roots)

✓ (Starting graph correctly)

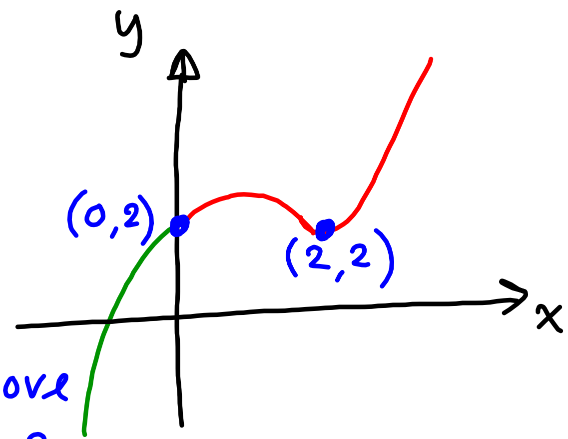
✓ (completing graph)

b) $y = 2 - h'(x)$

$y = -h'(x) + 2$

Reflect $y = h'(x)$ in the x-axis

then move UP by 2



✓ (0, 2)

✓ (2, 2)

✓ complete