Differentiation Non-Calculator HW

1)

A curve has equation $y = x - \frac{16}{\sqrt{x}}, \quad x > 0.$

Find the equation of the tangent at the point where $x = 4$. **6**

2)

Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of $45°$ with the positive direction of the x-axis. **4**

3)

Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

4)

Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$. **4**

5)

If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

6)

The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

(*a*) Find the value of x for which the gradient of the tangent at P is 12. **5**

(*b*) Hence find the equation of the tangent at P. **2**

7)

Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$. **3**

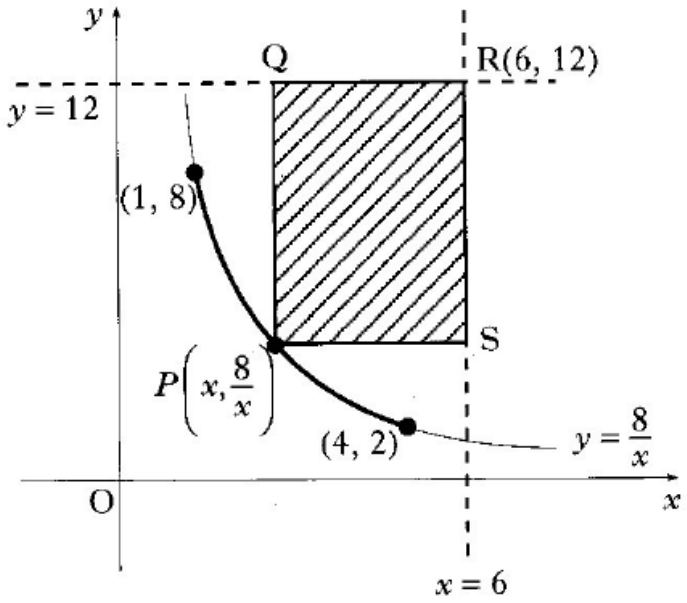
8)

A function f is defined by $f(x) = (2x - 1)^5$.
Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

9)

PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

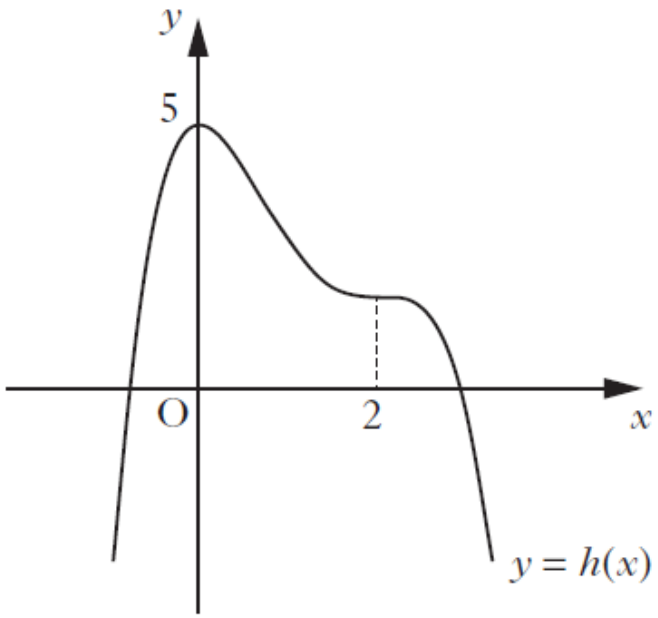
(ii) Hence show that the area, A square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}.$$ **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

10)

The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

(*a*) $y = h'(x)$; **3**

(*b*) $y = 2 - h'(x)$. **3**